NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

REPORT No. 519

SPINNING CHARACTERISTICS OF WINGS
I-RECTANGULAR CLARK Y MONOPLANE WING

By M. J. BAMBER and C. H. ZIMMERMAN

1935

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AERONAUTIC SYMBOLS

1. FUNDAMENTAL AND DERIVED UNITS

	Symbol	Metric		English	
		Unit	Abbreviation	Unit	Abbreviation
Length	l	meter	m	foot (or mile)	ft. (or mi.)
Time	t	second	s	second (or hour)	sec. (or hr.)
Force	F	weight of 1 kilogram	kg	weight of 1 pound	lb.
Power	P	horsepower (metric)		horsepower	hp.
Speed	V	kilometers per hour	k.p.h.	miles per hour	m.p.h.
		meters per second	m.p.s.	feet per second	f.p.s.

2. GENERAL SYMBOLS

W, Weight $= mg$

g, Standard acceleration of gravity $= 9.80665$ m/s^2 or 32.1740 ft./sec.^{2}

m, Mass $= \frac{W}{g}$

I, Moment of inertia $= mk^2$. (Indicate axis of radius of gyration k by proper subscript.)

μ, Coefficient of viscosity

ν, Kinematic viscosity

ρ, Density (mass per unit volume)

Standard density of dry air, 0.12497 kg-m^{-4}-s^2 at 15° C. and 760 mm; or 0.002378 lb.-ft.$^{-4}$ sec.^{2}

Specific weight of "standard" air, 1.2255 kg/m^3 or 0.07651 lb./cu.ft.

3. AERODYNAMIC SYMBOLS

S, Area

S_w, Area of wing

G, Gap

b, Span

c, Chord

$\frac{b^2}{S}$, Aspect ratio

V, True air speed

q, Dynamic pressure $= \frac{1}{2}\rho V^2$

L, Lift, absolute coefficient $C_L = \frac{L}{qS}$

D, Drag, absolute coefficient $C_D = \frac{D}{qS}$

D_o, Profile drag, absolute coefficient $C_{D_o} = \frac{D_o}{qS}$

D_i, Induced drag, absolute coefficient $C_{D_i} = \frac{D_i}{qS}$

D_p, Parasite drag, absolute coefficient $C_{D_p} = \frac{D_p}{qS}$

C, Cross-wind force, absolute coefficient $C_C = \frac{C}{qS}$

R, Resultant force

i_w, Angle of setting of wings (relative to thrust line)

i_t, Angle of stabilizer setting (relative to thrust line)

Q, Resultant moment

Ω, Resultant angular velocity

$\rho\frac{Vl}{\mu}$, Reynolds Number, where l is a linear dimension (e.g., for a model airfoil 3 in. chord, 100 m.p.h. normal pressure at 15° C., the corresponding number is 234,000; or for a model of 10 cm chord, 40 m.p.s. the corresponding number is 274,000)

C_p, Center-of-pressure coefficient (ratio of distance of *c.p.* from leading edge to chord length)

α, Angle of attack

ϵ, Angle of downwash

α_o, Angle of attack, infinite aspect ratio

α_i, Angle of attack, induced

α_a, Angle of attack, absolute (measured from zero-lift position)

γ, Flight-path angle

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NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

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SPINNING CHARACTERISTICS OF WINGS

I—RECTANGULAR CLARK Y MONOPLANE WING

By M. J. Bamber and C. H. Zimmerman

SUMMARY

A series of wind-tunnel tests of a rectangular Clark Y wing was made with the N. A. C. A. spinning balance as part of a general program of research on airplane spinning. All six components of the aerodynamic force and moment were measured throughout the range of angles of attack, angles of sideslip, and values of $\Omega b/2V$ likely to be attained by a spinning airplane; the results were reduced to coefficient form.

The latter part of the report contains an analysis illustrating the application of data from the spinning balance to an estimation of the angle of sideslip necessary for spinning equilibrium at any angle of attack. The analysis also shows the amount of yawing moment that must be supplied by the fuselage, tail, and interference effects in a steady spin. The effects of variation of such factors as mass distribution, attitude, wing loadings, etc., upon the likelihood of a monoplane with a rectangular Clark Y wing attaining a steady spin as revealed by the analysis are considered in the discussion.

It is concluded that a conventional monoplane with a rectangular Clark Y wing can be made to attain spinning equilibrium throughout a wide range of angles of attack but that provision of a yawing-moment coefficient of -0.02 (i. e. against the spin) by the tail, fuselage, and interferences will insure against attainment of equilibrium in a steady spin.

INTRODUCTION

Estimations of the probability of an airplane's attaining a steady spin and also of the ease and quickness of recovery can be made when the airplane is being designed only if data on the aerodynamic characteristics of the component parts, together with interference effects, are available for all spinning attitudes and conditions within the possible range. The National Advisory Committee for Aeronautics has undertaken an extensive program of research using the spinning balance to obtain such data for a number of wings and wing combinations. As rapidly as conditions permit the tests will be extended to cover tail and fuselage combinations and interference effects.

The first part of this report presents the aerodynamic characteristics of a rectangular Clark Y monoplane wing, which was the first wing tested on the spinning balance, throughout the ranges of angle of attack, angle of sideslip, and $\Omega b/2V$ likely to be attained in steady spins by an airplane of conventional type. In the second part of the report the data are analyzed to show the sideslip at the center of gravity and the yawing-moment coefficient necessary from parts of the airplane other than the wing for equilibrium in spins at various angles of attack for various loadings, mass distributions, and values of the pitching-moment coefficient.

The analysis illustrates the use of a method of estimating the effects of the wing characteristics upon the conditions necessary for steady spinning equilibrium. When sufficient data are available on the aerodynamic characteristics of various combinations of tails and fuselages throughout the spinning range, the method can be used to calculate actual spinning attitudes for specific combinations and, if extended, to estimate the time necessary for recovery from those attitudes with specific control movements. The method of analysis is similar to that developed by British investigators (reference 1) but differs from it in detail because of differences in the form of the available data.

AERODYNAMIC DATA

APPARATUS AND MODEL

Aerodynamic forces and moments in the various spinning attitudes were measured with the spinning balance (reference 2) in the N. A. C. A. 5-foot vertical wind tunnel (reference 3).

The Clark Y wing model is rectangular, 5 inches by 30 inches, with square tips. It is made of laminated mahogany and cut out at the center for a ball-clamp attachment to the balance. The model is shown in place on the spinning balance in figure 1.

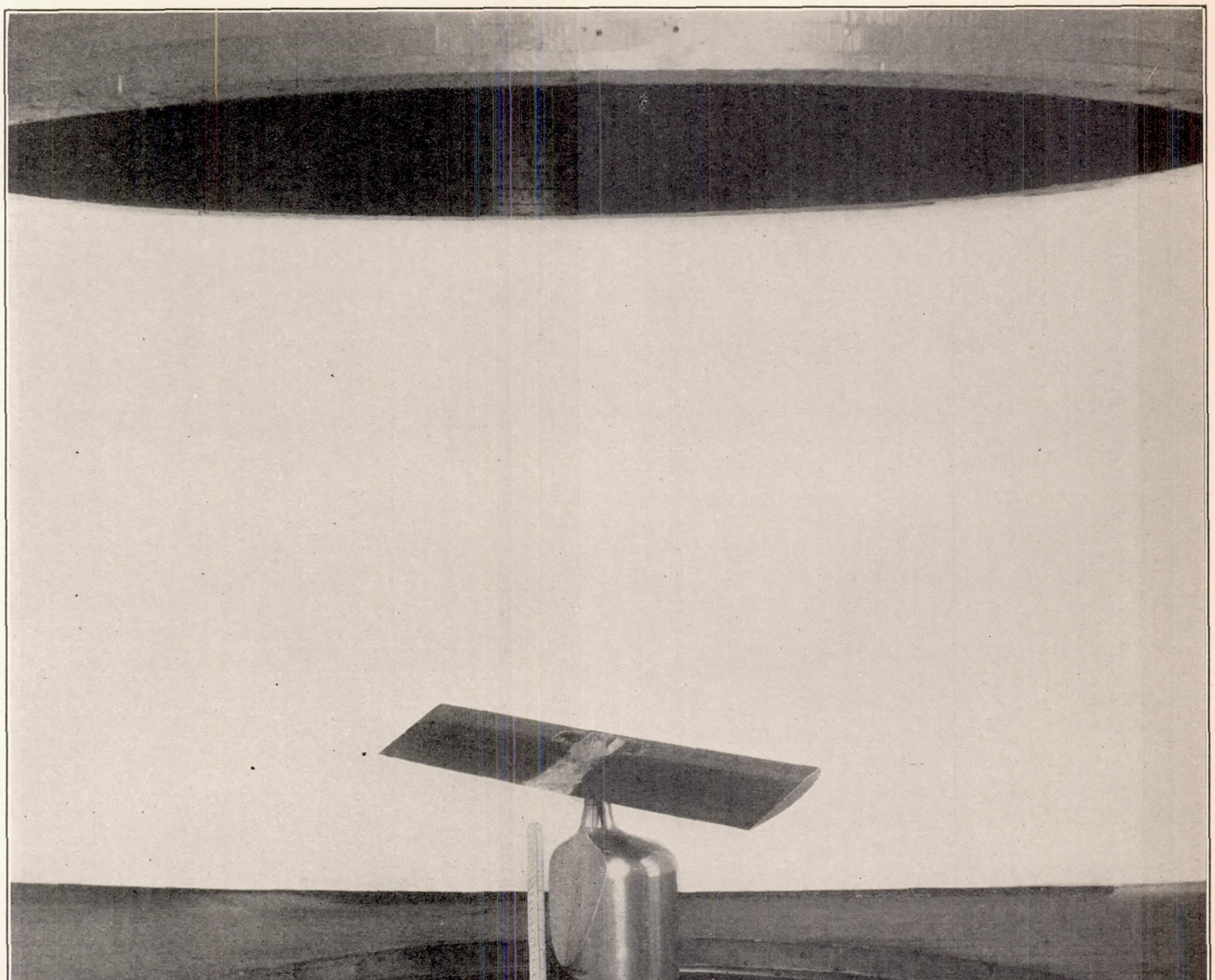

FIGURE 1.—The rectangular Clark Y wing mounted on the spinning balance in the vertical wind tunnel.

TESTS

All six components of the aerodynamic force and moment exerted on the model were measured at 5° intervals of angle of attack (measured at the plane of symmetry) from 25° to 70°. Because of the spread of the data, from 5 to 10 tests were made and the results averaged for each test condition at angles of attack of 30°, 40°, 50°, and 65°. At each angle of attack the model was tested with −10°, −5°, 0°, 5°, and 10° sideslip. At each angle of sideslip β for each angle of attack, tests were made with values of $\Omega b/2V$ of 0.25, 0.50, 0.75, and 1.00. All tests were made with the quarter-chord point of the lower surface of the wing at zero radius.

The air speed for values of $\Omega b/2V$ of 0.25 and 0.50 was 65 feet per second. With $\Omega b/2V$ equal to 0.75 and to 1.00 the air speeds were 60 and 45 feet per second, respectively. These speeds cover a range of values of Reynolds Number from 106,000 to 153,000. Early tests on the spinning balance indicated little scale effect within this range (reference 2).

SYMBOLS

The symbols used in the paper are listed here for ready reference.

α, Angle of attack at center of gravity.

$\beta=\sin^{-1}\frac{v}{V}$, Angle of sideslip at the center of gravity.

V, Resultant linear velocity of the center of gravity.

v, Linear velocity along the Y airplane axis, positive when the airplane is sideslipping to the right.

Ω, Resultant angular velocity, radians per second.

b, Span of wing.

S, Area of wing.

$q=\frac{1}{2}\rho V^2$, Dynamic pressure.

ρ, Air density, slug/cu. ft.

X, Longitudinal force acting along the X airplane axis, positive forward.

Y, Lateral force acting along the Y airplane axis, positive to the right.

Z, Normal force acting along the Z airplane axis, positive downward.

L, Rolling moment acting about the X airplane axis, positive when it tends to lower the right wing.

M, Pitching moment acting about the Y airplane axis, positive when it tends to increase the angle of attack.

N, Yawing moment acting about the Z airplane axis, positive when it tends to turn the airplane to the right.

Forces and moments with double primes (e. g., X'') are in the ground system of axes where Z'' is positive downward and X'' is along the radius of the spin, positive toward the center of the spin.

Coefficients of forces are obtained by dividing the force by qS.

Coefficients of moments are obtained by dividing the moment by qbS.

$\mu=\frac{m}{\rho Sb}$, Relative density of airplane to air. Under standard conditions, $\mu=13.1\ W/Sb$.

$m=W/g$, Mass.

k_X,k_Y,k_Z, Radii of gyration of the airplane about the X, Y, and Z airplane axes, respectively.

$\frac{b^2}{k_Z^2-k_X^2}=\frac{Wb^2}{g(C-A)}$, Pitching-moment inertia parameter.

$\frac{k_Z^2-k_Y^2}{k_Z^2-k_X^2}=\frac{C-B}{C-A}$, Rolling-moment and yawing-moment inertia parameter.

$A=mk_X^2$, Moment of inertia about the X airplane axis.

$B=mk_Y^2$, Moment of inertia about the Y airplane axis.

$C=mk_Z^2$, Moment of inertia about the Z airplane axis.

RESULTS

Results of the measurements have been reduced to the following coefficient forms, which are standard with the exception of that for pitching moment, for which the coefficient is based on the span rather than on the chord:

$$C_X=\frac{X}{qS} \qquad C_Y=\frac{Y}{qS} \qquad C_Z=\frac{Z}{qS}$$

$$C_l=\frac{L}{qbS} \qquad C_m=\frac{M}{qbS} \qquad C_n=\frac{N}{qbS}$$

Pitching-moment coefficients can be referred to the chord of the wing by multiplying the values given by 6. All values are given in terms of right spins.

The values of the coefficients for body axes are plotted against angle of attack in figures 2 to 7. The coefficients for ground axes (assuming the spin axis as the Z'' axis positive downward in flight) are plotted against angle of attack in figures 8 to 13.

Variations of C_l, C_m, and C_n with β and $\Omega b/2V$ are plotted for typical cases in figures 14 to 16.

The data given in the faired curves of figures 2 to 7 are believed to be correct for the model under the conditions of test within the following limits:

$C_X, \pm 0.01$ $\qquad$ $C_l, \pm 0.002$

$C_Y, \pm 0.01$ $\qquad$ $C_m, \pm 0.005$

$C_Z, \pm 0.04$ $\qquad$ $C_n, \pm 0.002$

No corrections have been made for tunnel-wall, blocking, or scale effects.

DISCUSSION

Although values of the coefficients have been included for ground axes, in order to avoid confusion this discussion will be confined to the coefficients based on body axes.

Longitudinal-force coefficient C_X.—The longitudinal-force coefficient was small throughout the range. In

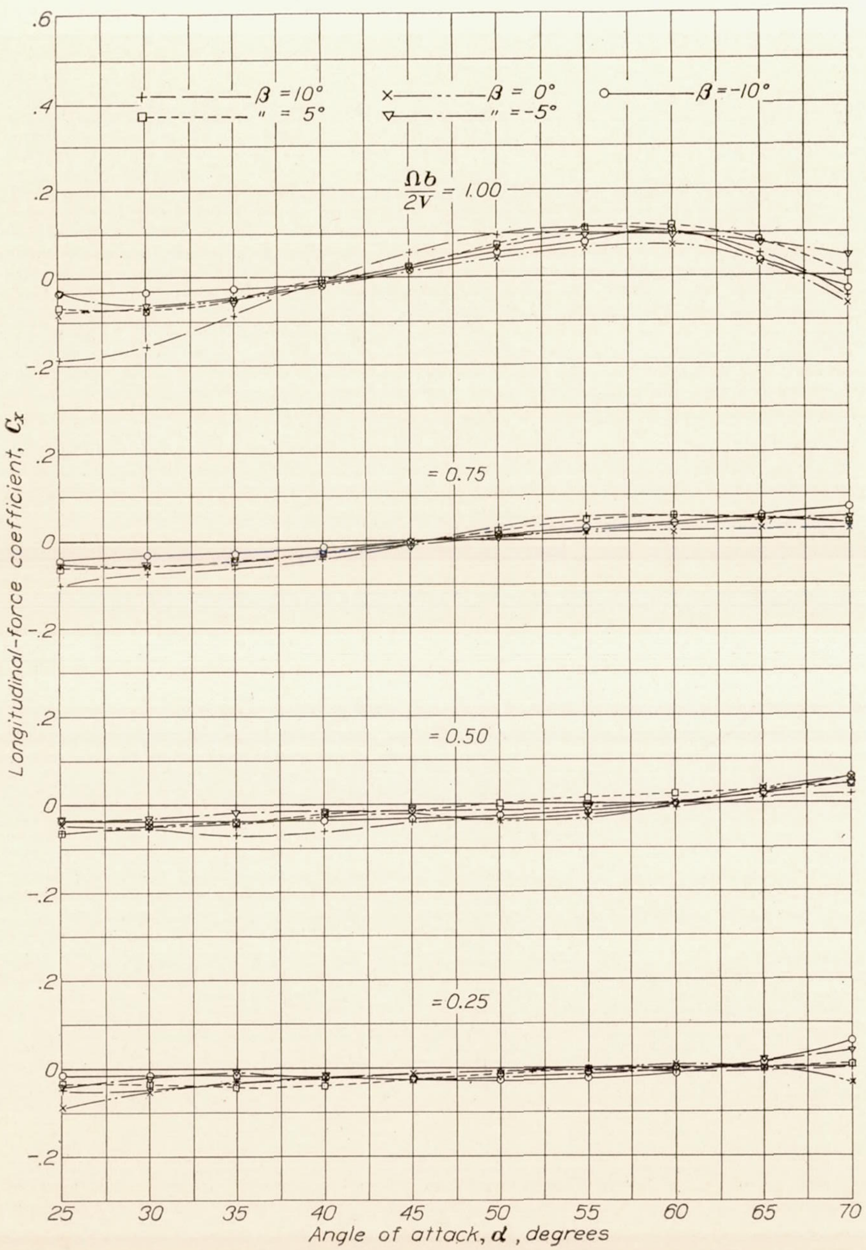

FIGURE 2.—Variation of longitudinal-force coefficient C_X (body axes) with angle of attack.

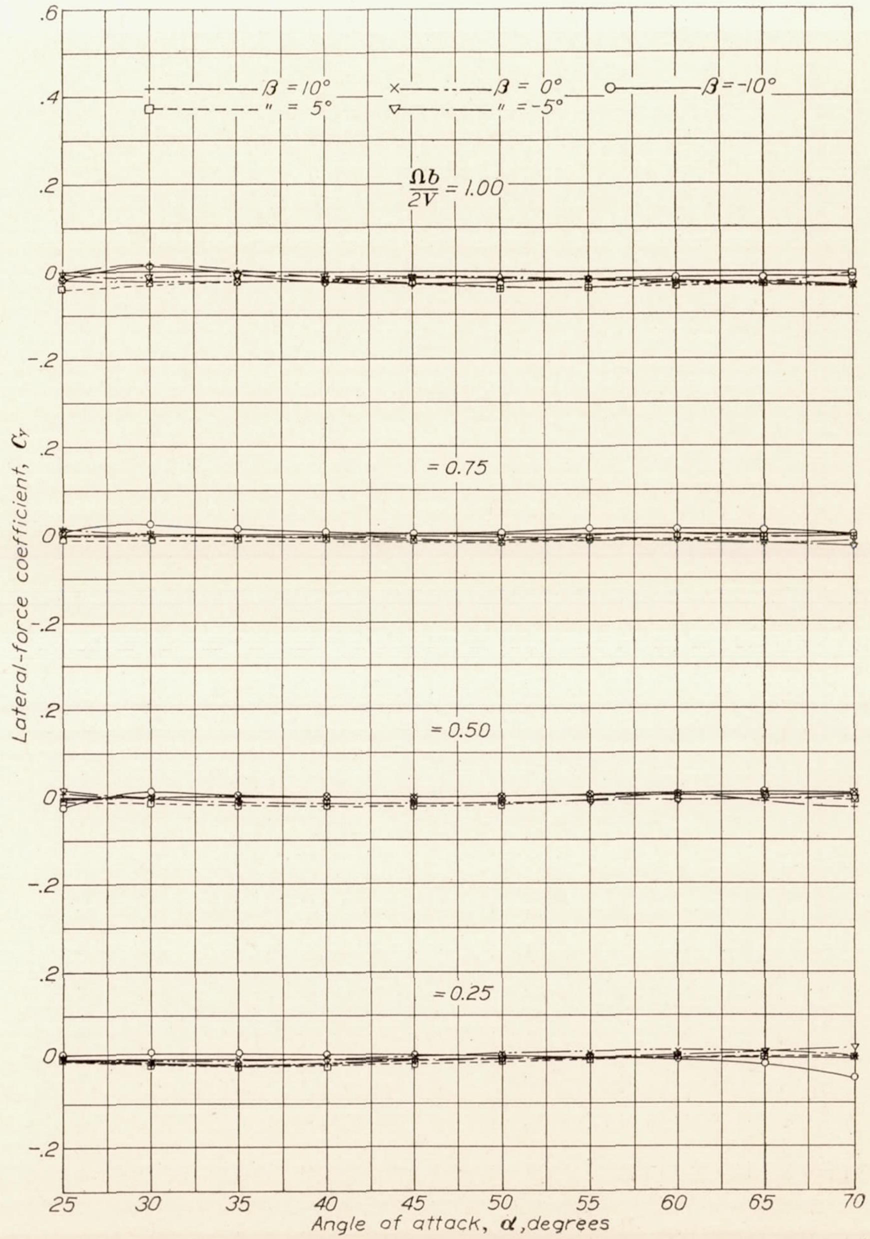

FIGURE 3.—Variation of lateral-force coefficient C_Y (body axes) with angle of attack.

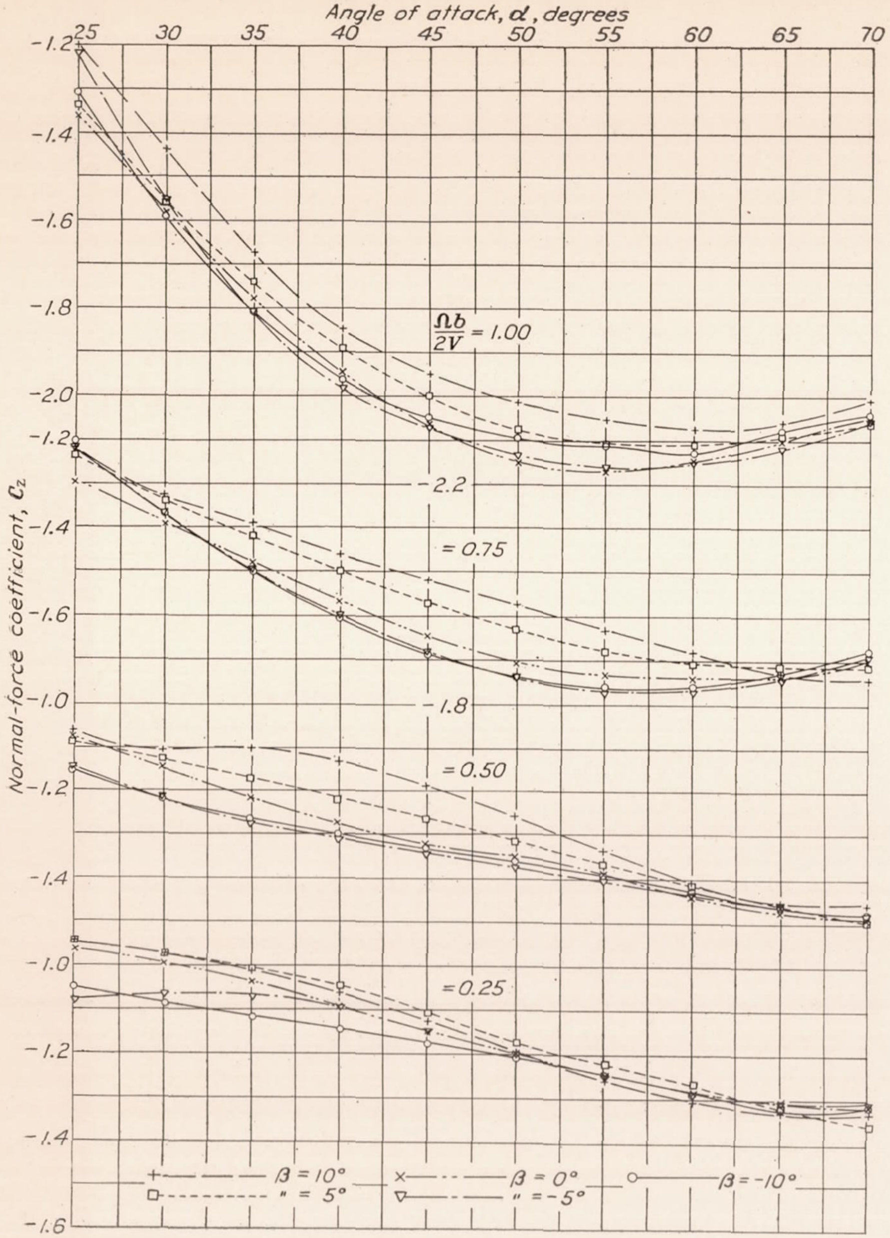

FIGURE 4.—Variation of normal-force coefficient C_Z (body axes) with angle of attack.

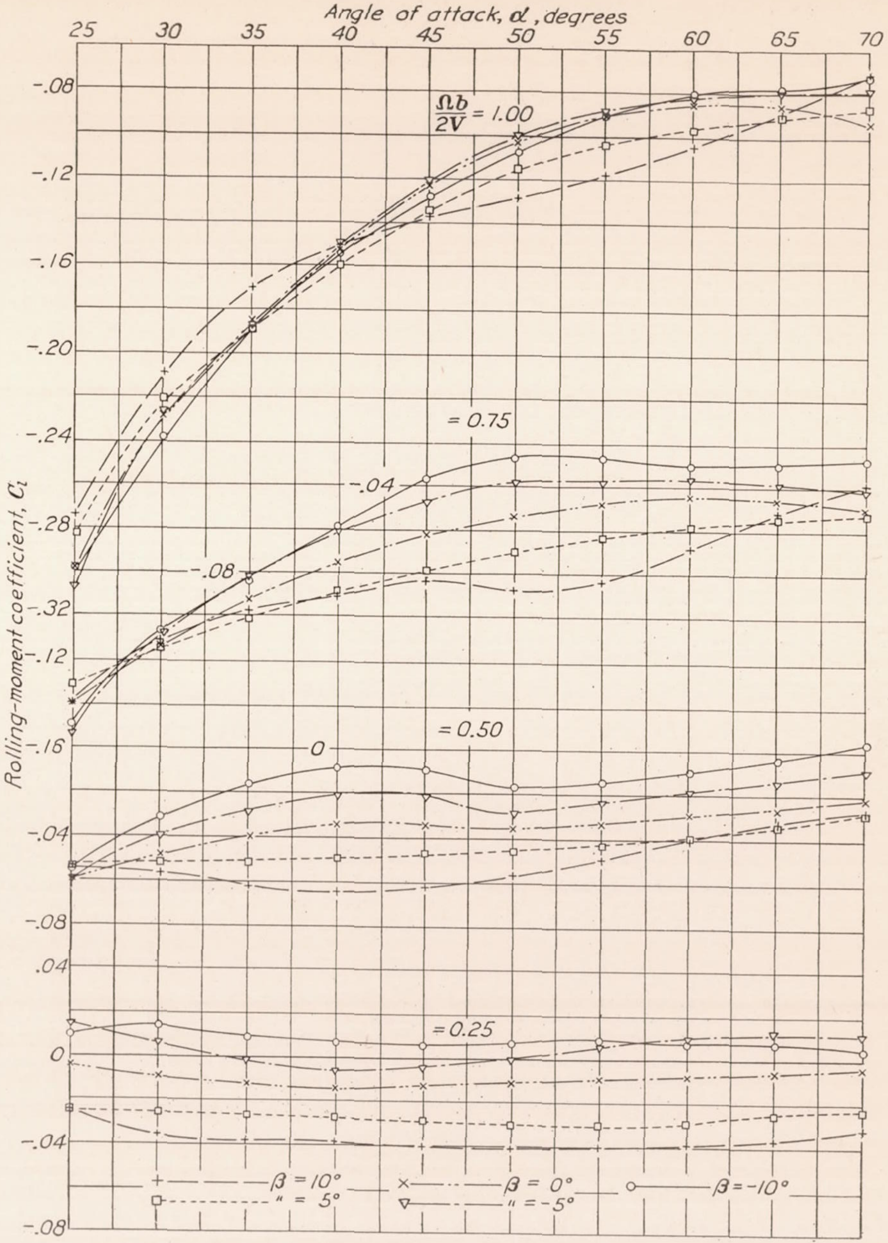

FIGURE 5.—Variation of rolling-moment coefficient C_l (body axes) with angle of attack.

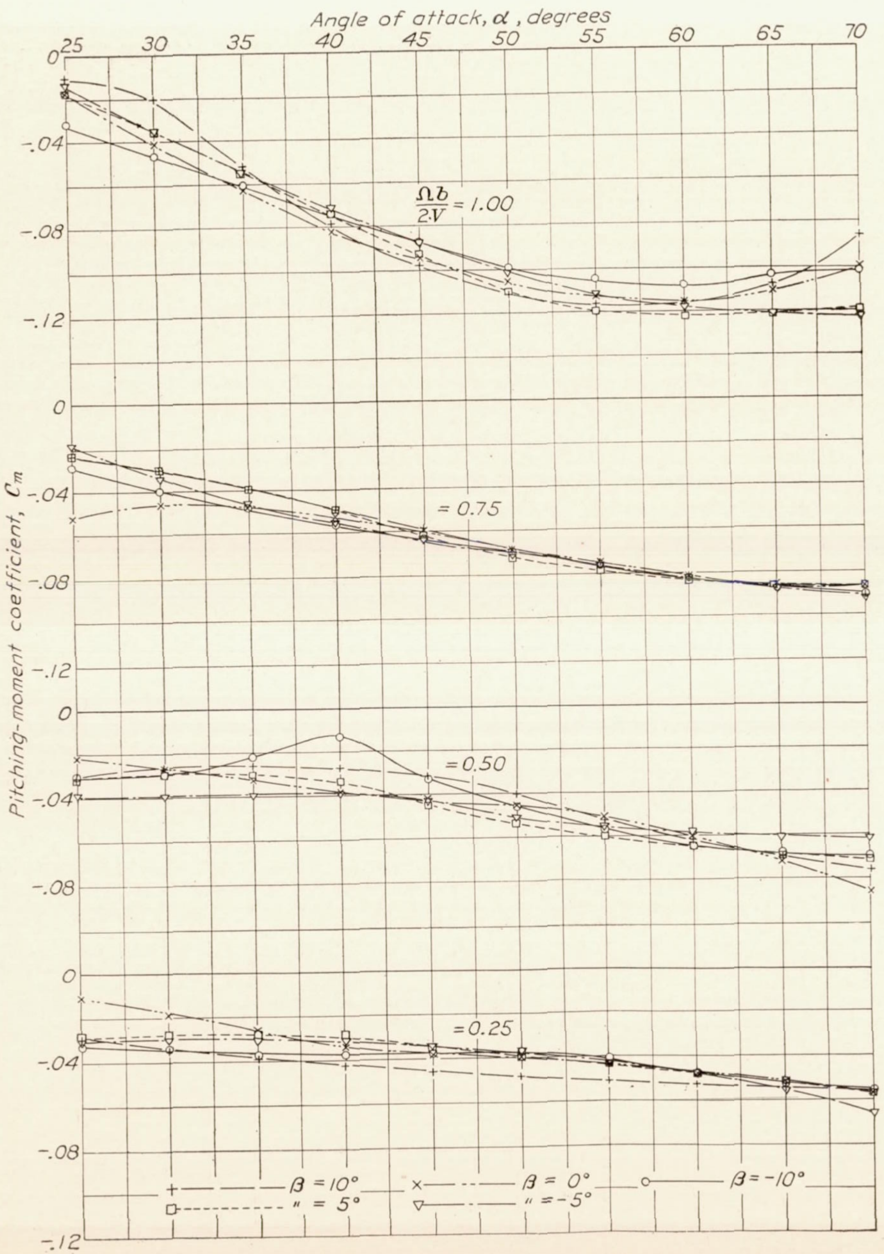

FIGURE 6.—Variation of pitching-moment coefficient C_m (body axes) with angle of attack.

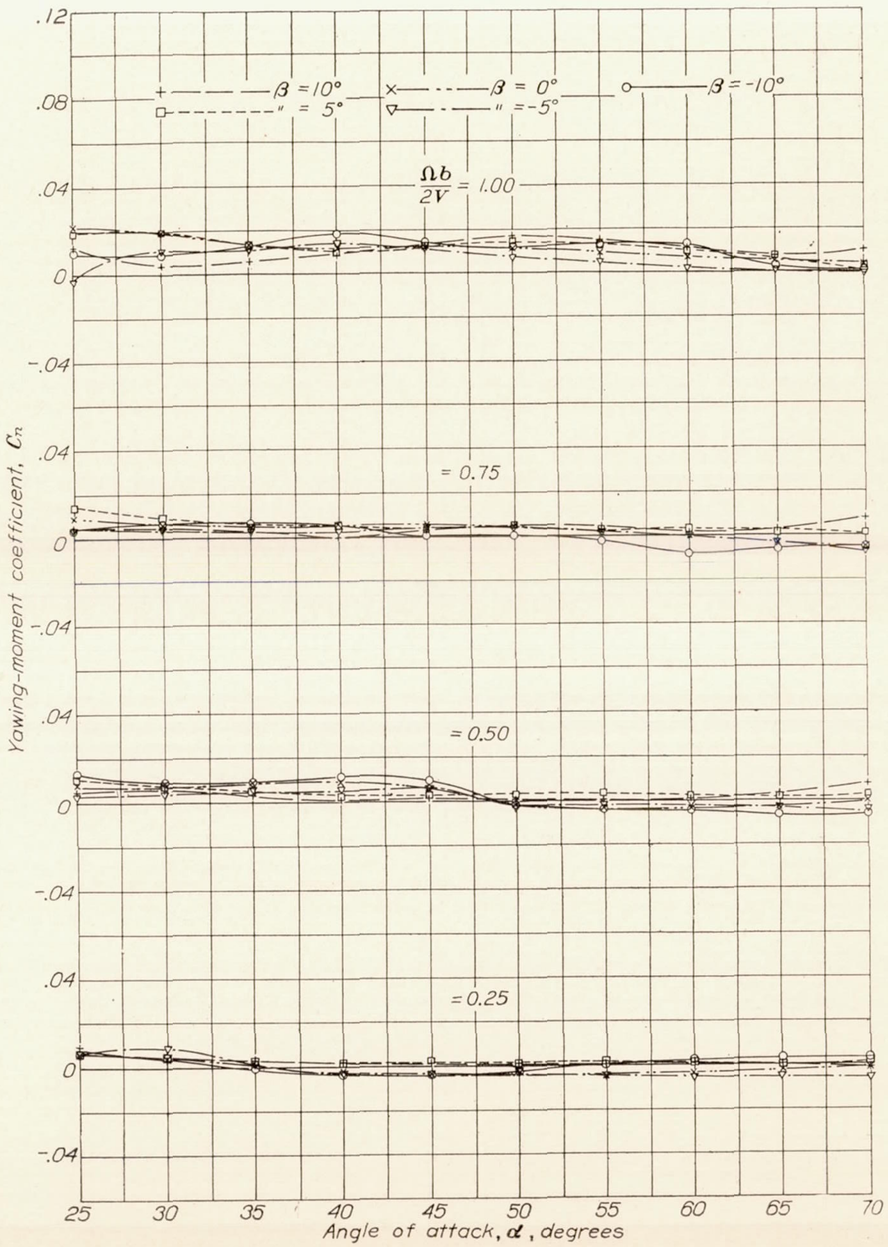

FIGURE 7.—Variation of yawing-moment coefficient C_n (body axes) with angle of attack.

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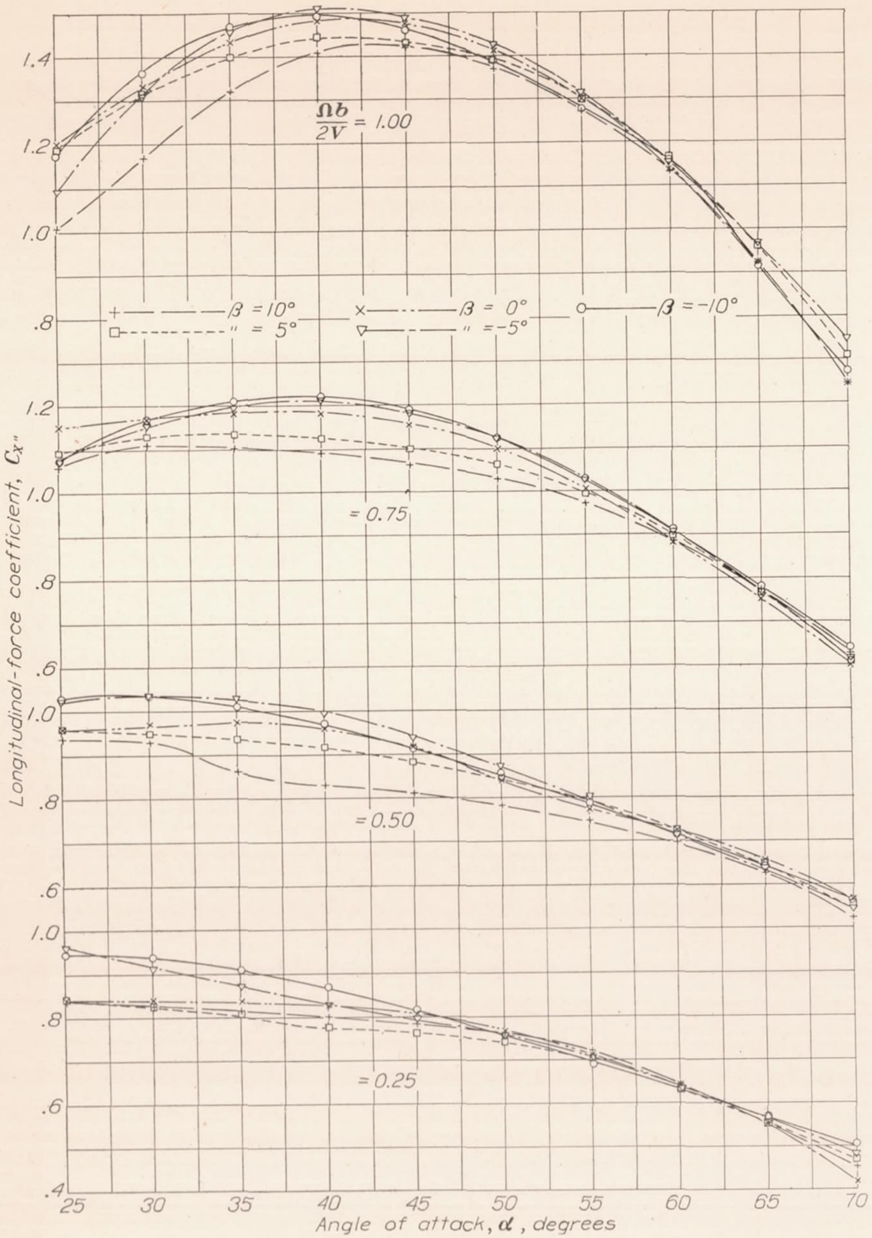

FIGURE 8.—Variation of longitudinal-force coefficient C_X'' (ground axes) with angle of attack.

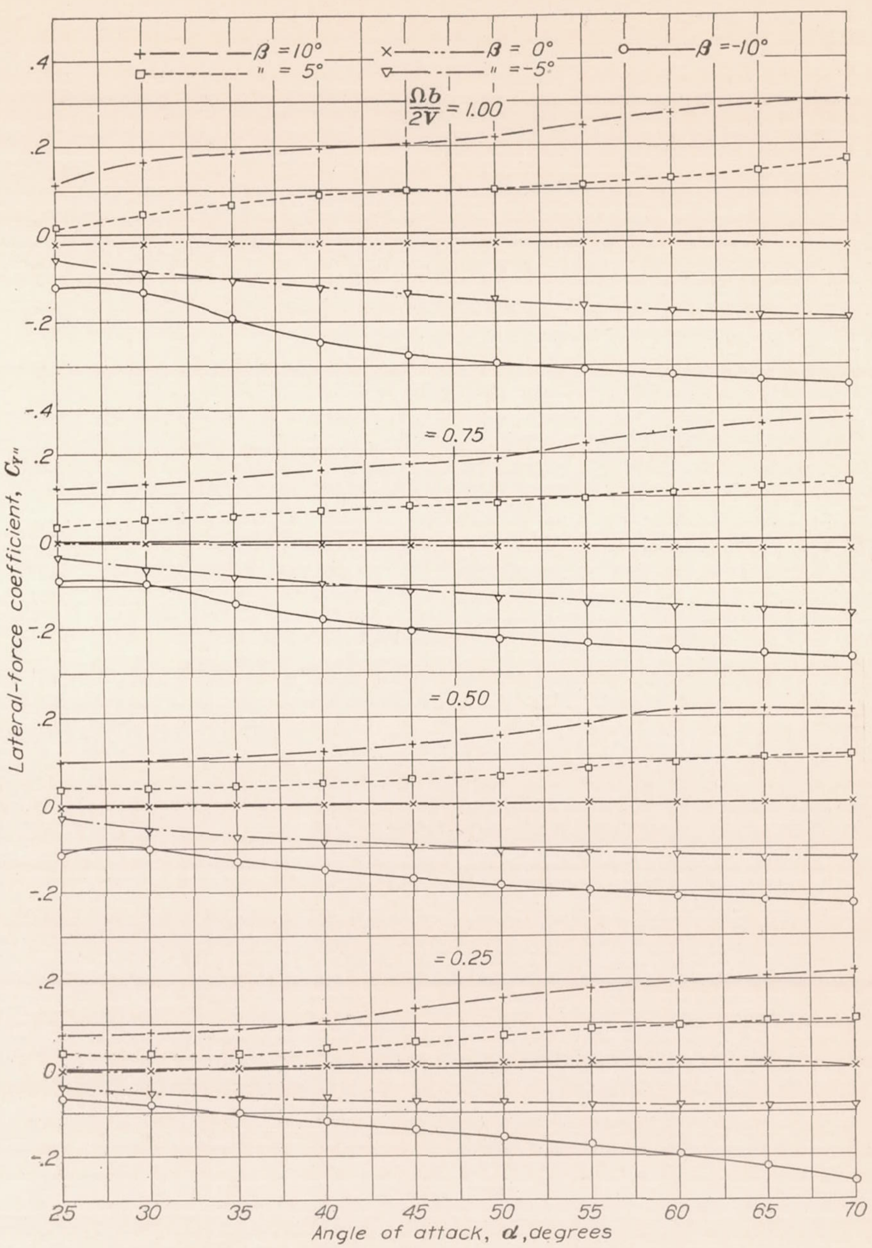

FIGURE 9.—Variation of lateral-force coefficient C_Y'' (ground axes) with angle of attack.

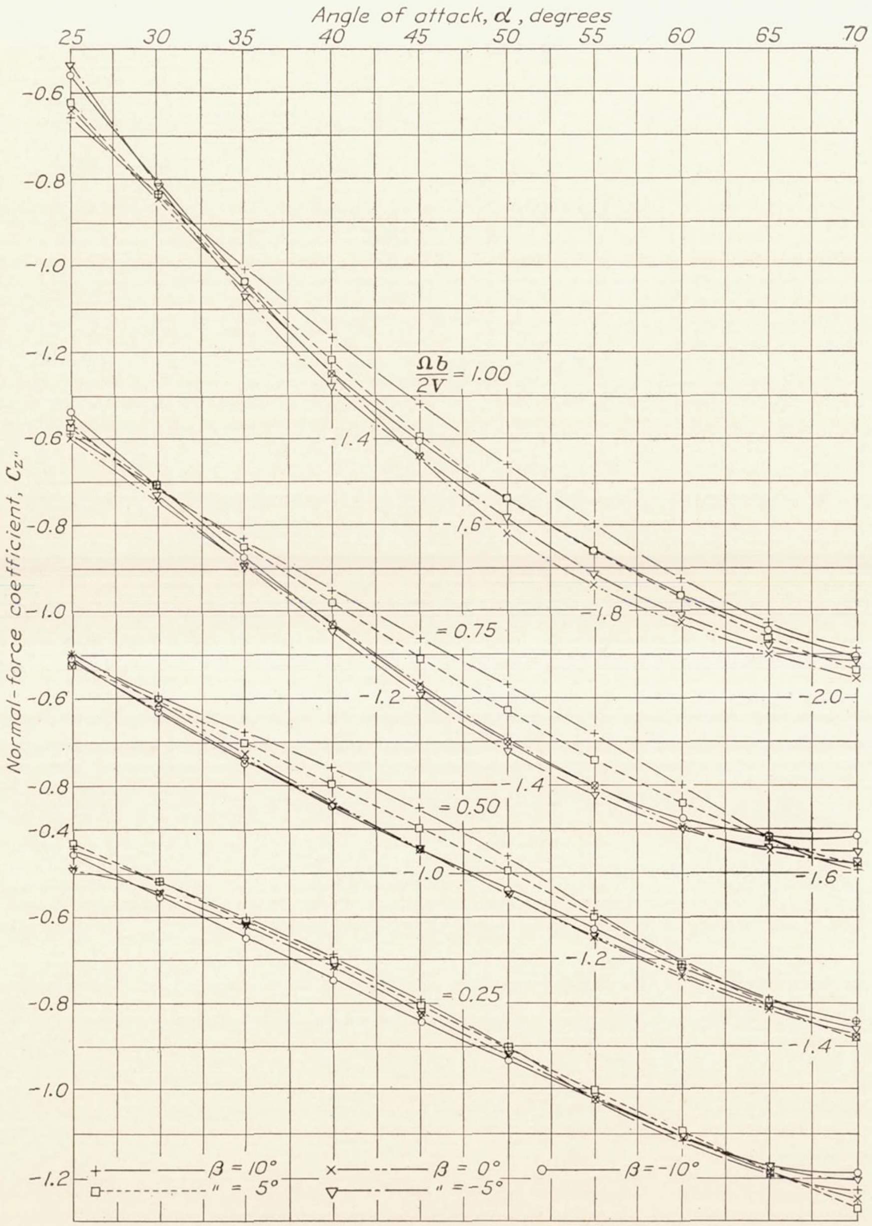

FIGURE 10.—Variation of normal-force coefficient C_Z'' (ground axes) with angle of attack.

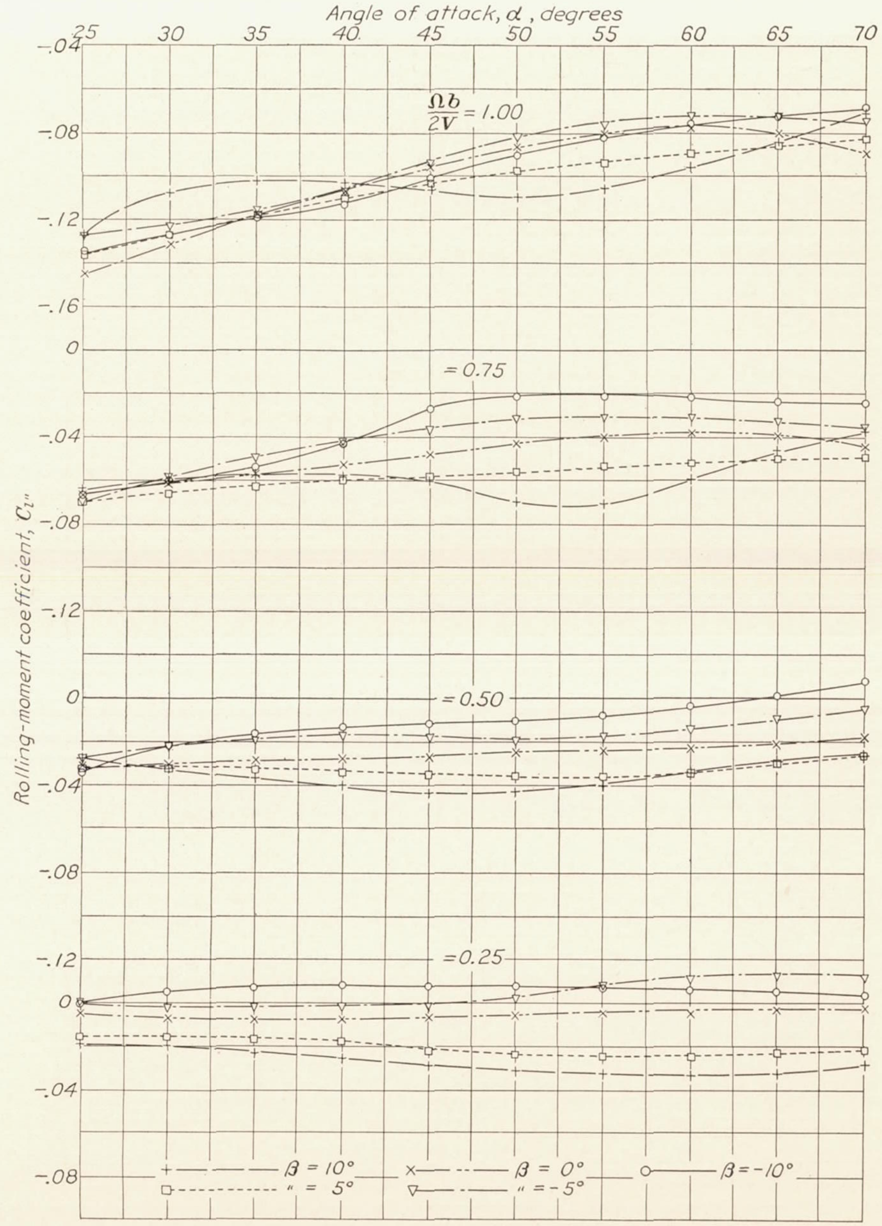

FIGURE 11.—Variation of rolling-moment coefficient C_l'' (ground axes) with angle of attack.

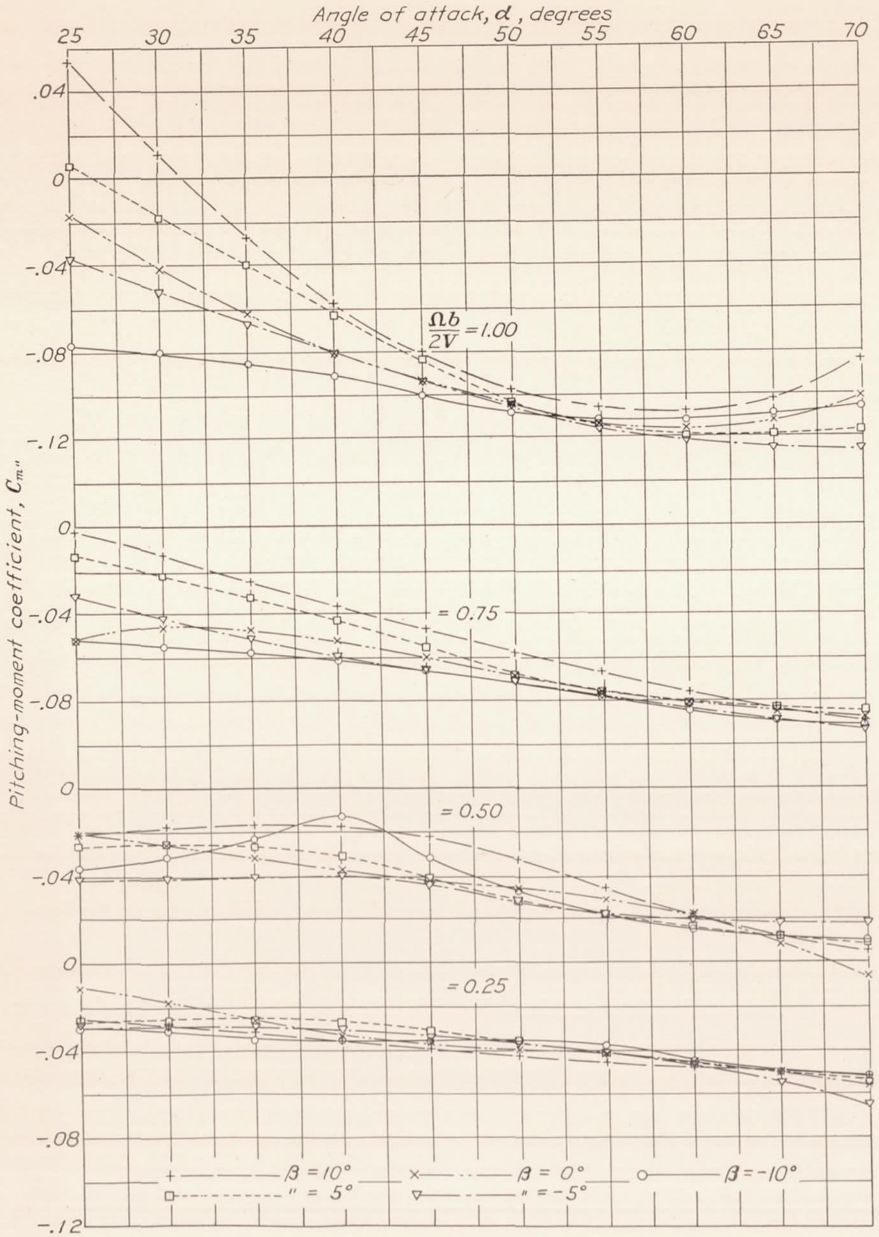

FIGURE 12.—Variation of pitching-moment coefficient C_m'' (ground axes) with angle of attack.

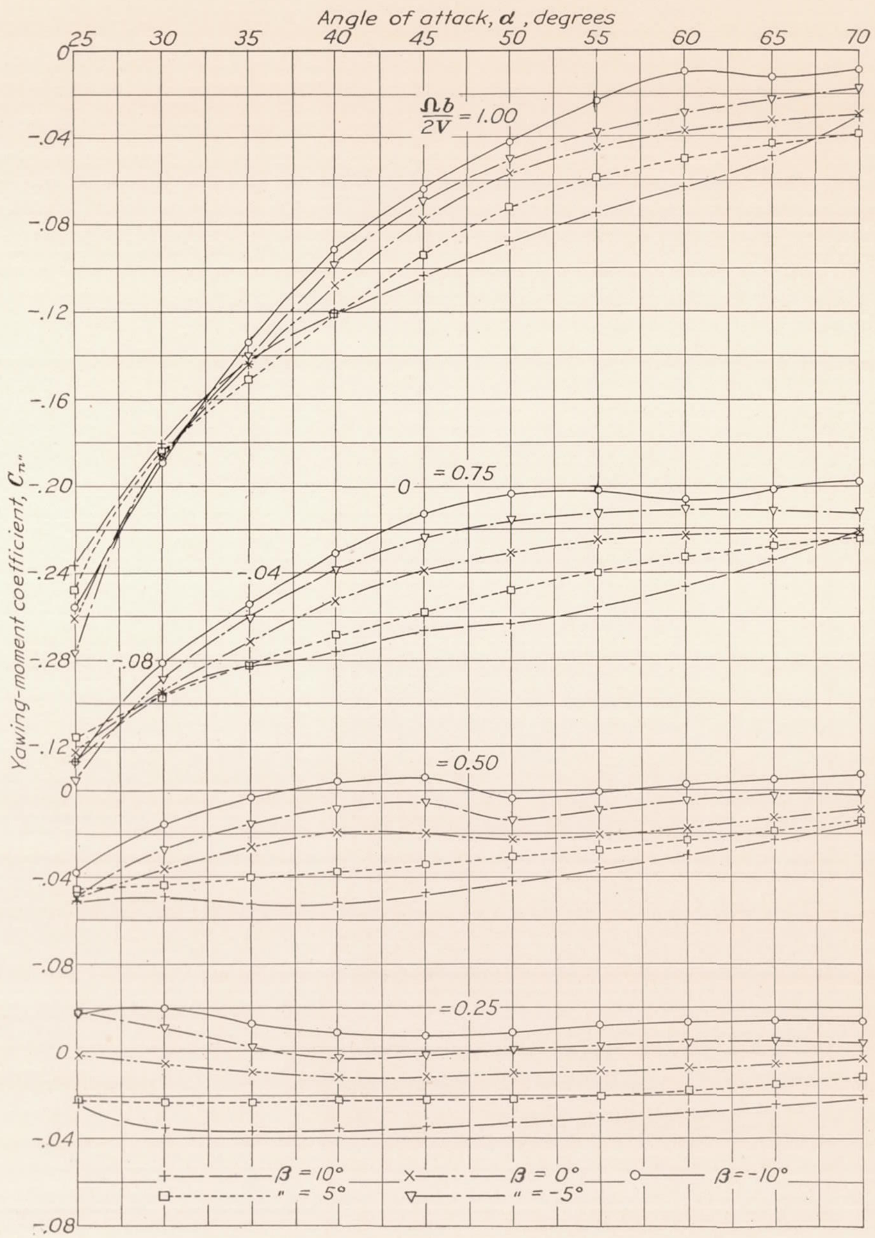

FIGURE 13.—Variation of yawing-moment coefficient C_n'' (ground axes) with angle of attack.

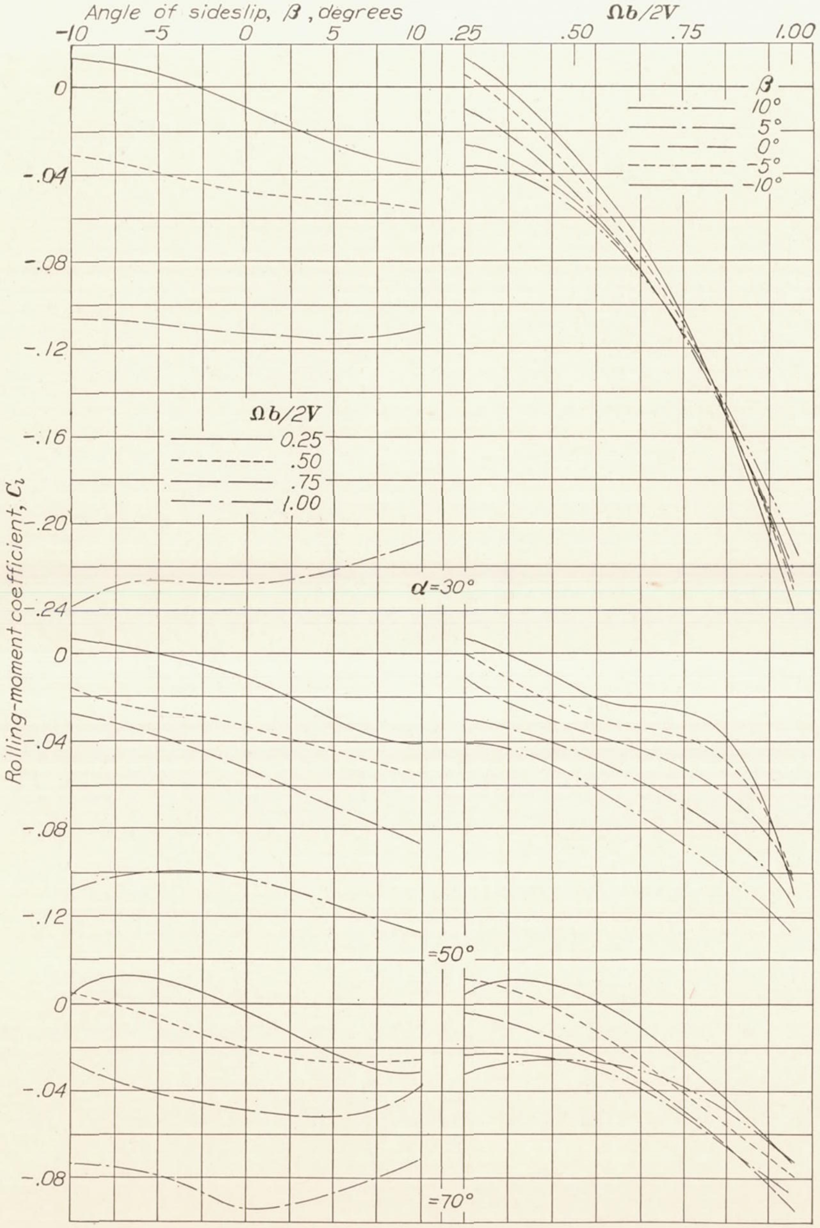

FIGURE 14.—Variation of rolling-moment coefficient C_l (body axes) with angle of sideslip and with $\Omega b/2V$.

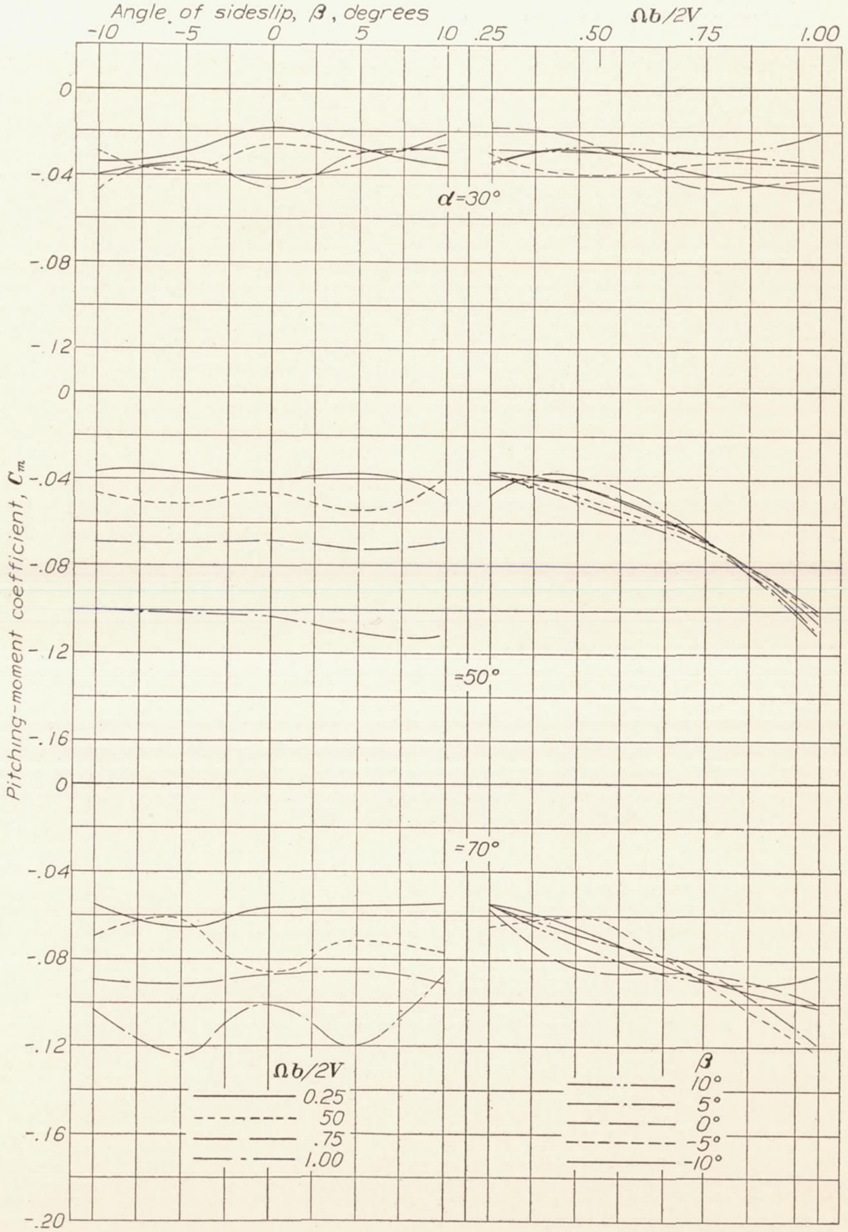

FIGURE 15.—Variation of pitching-moment coefficient C_m (body axes) with angle of sideslip and with $\Omega b/2V$.

general, values were negative at low angles of attack and positive at high angles. Changes with sideslip were small and irregular.

Lateral-force coefficient C_Y.—The lateral-force coefficient was very small, never greater than 0.05 in absolute value, and no decided trends were indicated.

Normal-force coefficient C_Z.—The normal-force vector was nearly the same as the resultant-force vector throughout the tests. It increased in magnitude with angle of attack except at the higher values of $\Omega b/2V$ between the angles of 55° and 70°. The change with $\Omega b/2V$ was quite marked. The highest value of the coefficient reached was 2.17 at $\alpha=55°$ with $\Omega b/2V=1.00$. This value is 43 percent greater than that predicted by the strip method, assuming a rectangular distribution. The normal force changed considerably with sideslip, particularly at the higher values of $\Omega b/2V$. In general, there was but small change with outward (negative) sideslip but definite decrease in value with inward sideslip.

Rolling-moment coefficient C_l.—The rolling-moment coefficient changed but slightly with angle of attack at low values of $\Omega b/2V$. At values of $\Omega b/2V$ greater than 0.50 the decrease in moment opposing the rotation with increase in angle of attack was quite marked. Sideslip had a pronounced effect upon the rolling-moment coefficient. With $\Omega b/2V=0.25$, the coefficient became more positive with change of sideslip from inward to outward at all angles of attack. The same was true with $\Omega b/2V=0.50$ except at an angle of attack of 25°, where the change was slight and indefinite. At values of $\Omega b/2V$ of 0.75 and 1.00, the change of rolling-moment coefficient with change in sideslip was the same as at the lower rates of rotation at intermediate angles of attack but changed sign at each end of the angle-of-attack range. Increasing the value of $\Omega b/2V$ resulted in larger negative values of C_l throughout the range of the tests, the effect being more pronounced at low angles of attack than at high angles.

Pitching-moment coefficient C_m.—The pitching-moment coefficient became more negative with increase in angle of attack at all values of β and $\Omega b/2V$. The curves are similar to those of C_Z. Pitching-moment coefficient changed irregularly with sideslip. In general, the changes were small and revealed no distinct trends. At 30° angle of attack no definite trend of change of C_m with $\Omega b/2V$ was indicated. At higher angles of attack C_m became more negative with increase in $\Omega b/2V$.

Yawing-moment coefficient C_n.—Values of yawing-moment coefficient were small throughout the range covered by the tests, the maximum value being 0.022 with $\alpha=25°$, $\beta=0°$, and $\Omega b/2V=1.00$. There was a general tendency for C_n to decrease with increase in angle of attack, although individual curves had positive slopes at some angles of attack. Changes of C_n with sideslip were small and irregular and did not reveal any definite trends. Yawing-moment coefficient curves revealed a general tendency for the coefficient to increase with increase of $\Omega b/2V$, although individual curves had negative slopes over part of the $\Omega b/2V$ range.

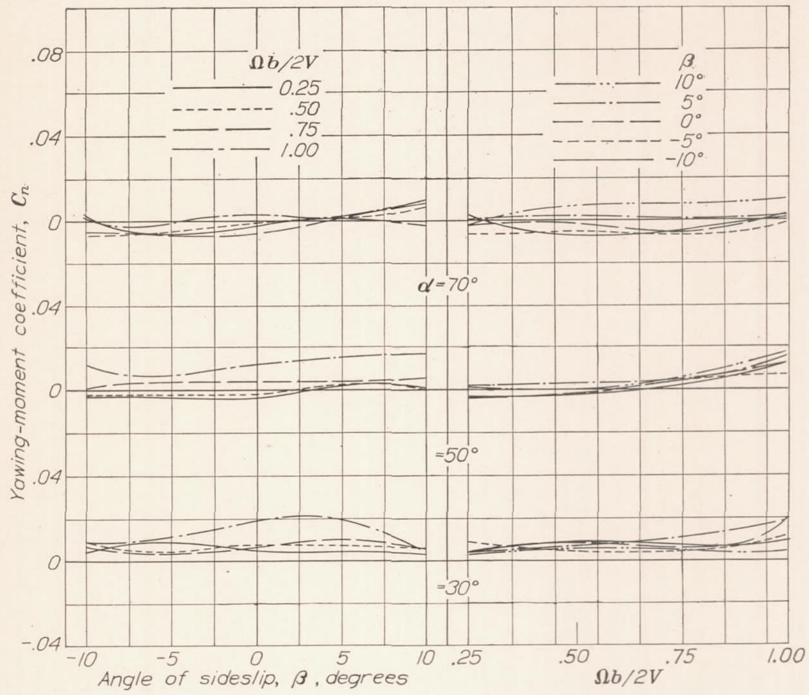

FIGURE 16.—Variation of yawing-moment coefficient C_n (body axes) with angle of sideslip and with $\Omega b/2V$.

ANALYSIS OF SPINNING CHARACTERISTICS

FORMULAS FOR COMPUTATION OF SPINNING EQUILIBRIUM OF AIRPLANES

The following formulas were obtained from exact equations for equilibrium in a steady spin by a series of approximations. (See Appendix.) Comparison of values of C_l for equilibrium estimated by this method with values obtained from flight results (references 4 and 5) shows the results to fall well within the experimental accuracy of the aerodynamic data for ordinary values of sideslip. For values of sideslip of 10° inward or 15° outward the results may be in error by as much as 10 percent, the estimated values being less in absolute magnitude than the true values. From a practical standpoint even this amount of discrepancy is insignificant, the only effect being a slightly erroneous estimation of the amount of sideslip necessary for equilibrium and a small error in estimating the gyroscopic yawing-moment coefficient.

The formulas are:

$$\frac{\Omega b}{2V}=\sqrt{\frac{-C_m}{3.84\mu \sin 2\alpha}\times\frac{b^2}{k_Z^2-k_X^2}} \tag{1}$$

$$C_l=C_L\left(\frac{k_Z^2-k_Y^2}{b\sqrt{k_Z^2-k_X^2}}\right)\sqrt{\frac{-C_m \tan \alpha}{2\mu}}$$

$$+1.02\left(\frac{k_Z^2-k_Y^2}{k_Z^2-k_X^2}\right)\frac{-C_m \sin \beta}{\cos \alpha} \tag{2}$$

$$C_n=C_l \cot \alpha\left(\frac{k_Y^2-k_X^2}{k_Z^2-k_Y^2}\right) \tag{3}$$

where $\mu=\frac{m}{\rho S b}$. Under standard conditions, $\mu=\frac{13.1 W}{S b}$ (reference 6).

RANGE OF INDEPENDENT VARIABLES USED IN COMPUTATIONS

Estimations were made of the spinning equilibrium from 30° to 70° angle of attack and from −10° to 10° angle of sideslip for a representative case with the following assumed characteristics:

Slope of pitching-moment curve, $\frac{-C_m}{\alpha-20°}=0.0020$

Lift coefficient, $C_L=C_X''$

Relative density of airplane to air, $\mu=5.0$

Pitching-moment inertia parameter,

$$\frac{b^2}{k_Z^2-k_X^2}=80$$

Rolling-moment and yawing-moment inertia parameter,

$$\frac{k_Z^2-k_Y^2}{k_Z^2-k_X^2}=1.0$$

and for 15 additional cases determined by changing the variables one at a time from this mean. These conditions are summarized in table I.

TABLE I
CONDITIONS USED AS BASIS OF SPINNING ESTIMATIONS

$\frac{-C_m}{\alpha-20°}$	C_L	μ	$\frac{b^2}{k_Z^2-k_X^2}$	$\frac{k_Z^2-k_Y^2}{k_Z^2-k_X^2}$
0.0010	C_X''	5.0	80	1.0
.0015	C_X''	5.0	80	1.0
.0020	C_X''	5.0	80	1.0
.0025	C_X''	5.0	80	1.0
.0030	C_X''	5.0	80	1.0
.0020	$0.8C_X''$	5.0	80	1.0
.0020	$1.2C_X''$	5.0	80	1.0
.0020	C_X''	2.5	80	1.0
.0020	C_X''	7.5	80	1.0
.0020	C_X''	10.0	80	1.0
.0020	C_X''	5.0	60	1.0
.0020	C_X''	5.0	100	1.0
.0020	C_X''	5.0	120	1.0
.0020	C_X''	5.0	80	.5
.0020	C_X''	5.0	80	1.5
.0020	C_X''	5.0	80	2.0

The variations in relative density μ (reference 6) include the range of conventional airplanes, $\mu=2.5$ corresponding to a wing loading of 6 pounds per square foot and a span of 31.2 feet and $\mu=10$ corresponding to a wing loading of 20 with a span of 26.1 feet, under standard conditions.

Variations in the parameters

$$\frac{b^2}{k_Z^2-k_X^2} \text{ and } \frac{k_Z^2-k_Y^2}{k_Z^2-k_X^2}$$

cover the range given in reference 7 for 11 airplanes. These parameters may be rewritten as

$$\frac{Wb^2}{g(C-A)} \text{ and } \frac{C-B}{C-A}, \text{ respectively,}$$

where $A=mk_X^2$, the moment of inertia about the X axis.

$B=mk_Y^2$, the moment of inertia about the Y axis.

$C=mk_Z^2$, the moment of inertia about the Z axis.

METHOD OF ARRIVING AT CONDITIONS FOR SPINNING EQUILIBRIUM

The values of angle of sideslip and of aerodynamic yawing-moment coefficient that must be contributed by parts of the airplane other than the wing were estimated by the following method:

The value of $\Omega b/2V$ was computed with the appropriate values of C_m, μ, and $\frac{b^2}{k_Z^2-k_X^2}$, using equation (1) for angles of attack of 30°, 40°, 50°, 60°, and 70°.

The aerodynamic rolling-moment coefficient necessary for equilibrium was computed from equation (2), using appropriate values of C_L, $\frac{k_Z^2-k_Y^2}{b\sqrt{k_Z^2-k_X^2}}$, $\frac{k_Z^2-k_Y^2}{k_Z^2-k_X^2}$, μ, and C_m for angles of sideslip of −10°, −5°, 0°, 5°, and 10° at angles of attack of 30°, 40°, 50°, 60°, and 70°. The value of C_L was obtained from the test data for the appropriate values of α, β, and $\Omega b/2V$. The value of C_m was assumed not to change with sideslip. Values of C_l so computed were plotted against angle of sideslip. (See figs. 17 and 18.)

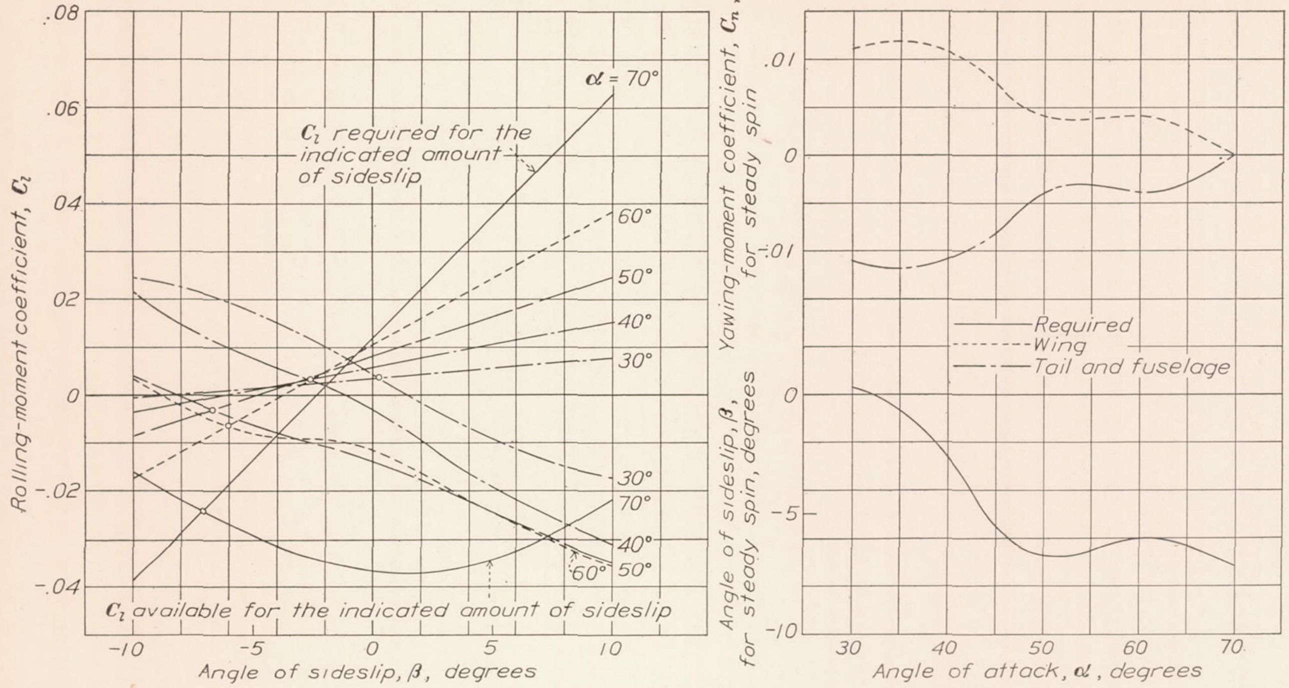

FIGURE 17.—Sample chart illustrating method of determining angle of sideslip and tail and fuselage yawing-moment coefficients necessary for equilibrium in spins with $k_X^2=k_Y^2$.

$C_m=-0.0020(\alpha-20°)$ $\mu=10$ $C_L=C_X''$

$\frac{k_Z^2-k_Y^2}{k_Z^2-k_X^2}=1.0$ $\frac{b^2}{k_Z^2-k_X^2}=80$

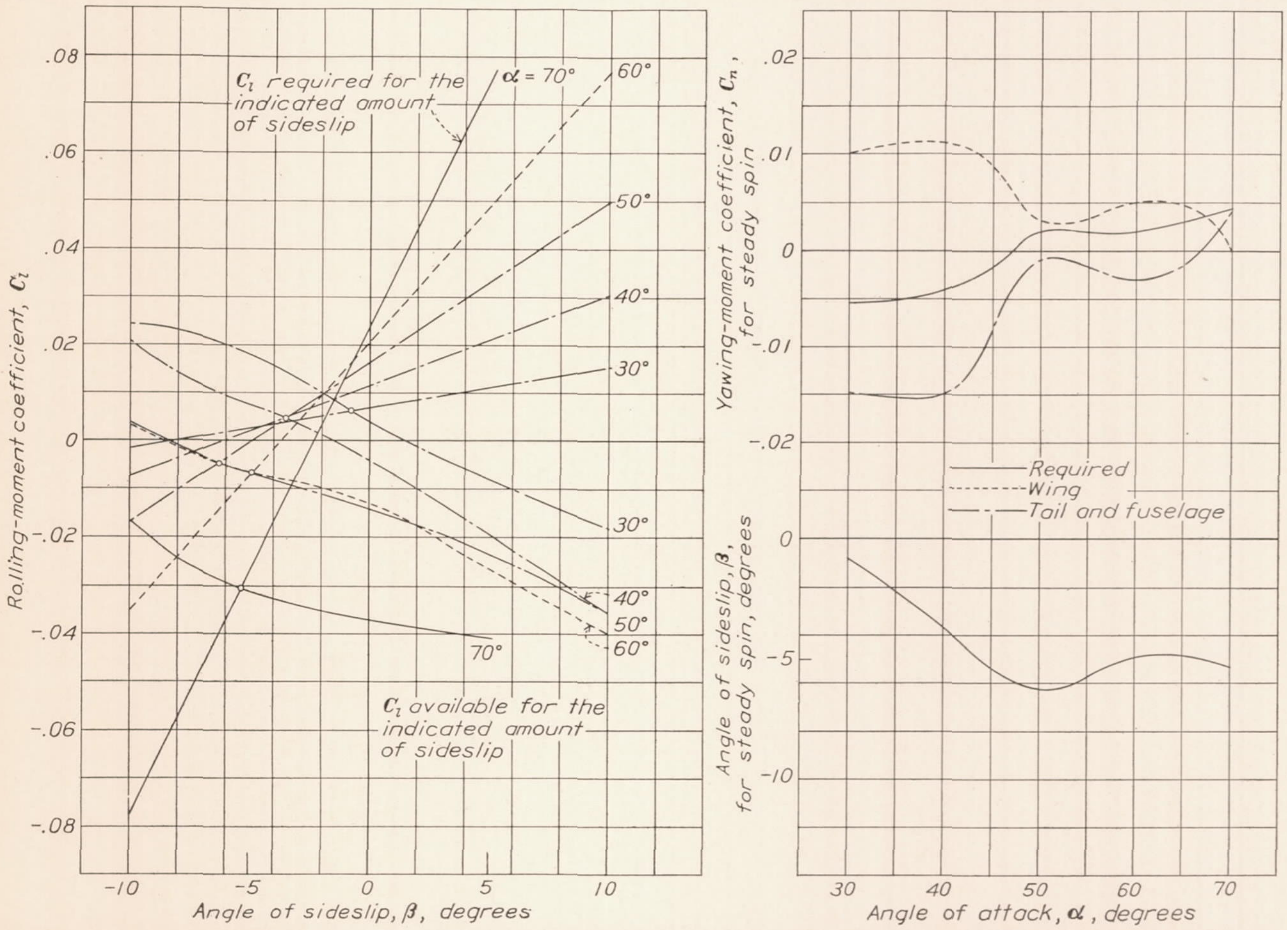

FIGURE 18.—Sample chart illustrating method of determining angle of sideslip and tail and fuselage yawing-moment coefficients necessary for equilibrium in spins with $k_X^2>k_Y^2$.

$C_m=-0.0020(\alpha-20°)$ $\mu=10$ $C_L=C_X''$

$\frac{k_Z^2-k_Y^2}{k_Z^2-k_X^2}=2.0$ $\frac{b^2}{k_Z^2-k_X^2}=80$

Aerodynamic values of C_l were determined for the values of $\Omega b/2V$ computed from equation (1) at the appropriate values of α and β by adding to the value given by the wing on the spinning balance an arbitrary constant amount $\Delta C_l=0.02$, for reasons given in the discussion. These values of C_l were also plotted in figures 17 and 18. The points at which curves for the same angle of attack intersect represent conditions of equilibrium of all forces and moments except yawing moment. A smooth curve was drawn connecting such points.

Values of C_n necessary to balance the gyroscopic yawing moment were calculated from equation (3) for the values of C_l on the curve of equilibrium of rolling moments and were plotted against angle of attack. The values of C_n for the appropriate conditions of α, β, and $\Omega b/2V$ from the tests of the Clark Y wing were increased by $\Delta C_n=0.006$ and plotted against α on the same charts. The algebraic difference between the two curves of C_n already plotted was then plotted and represents the value of C_n that must be supplied by fuselage, empennage, interference effects, etc., to give equilibrium in a steady spin at the given angle of attack.

RESULTS OF COMPUTATIONS

Sample charts illustrating the method of estimating the value of C_l and of sideslip for rolling-moment equilibrium at any angle of attack and giving the variation of sideslip with angle of attack for those particular cases, are given in figures 17 and 18. Sample variations of C_n with angle of attack are also shown.

The variations of sideslip necessary for equilibrium in a spin with $\frac{-C_m}{\alpha-20°}$, μ, C_L, $\frac{b^2}{k_X^2-k_Z^2}$, and $\frac{k_Z^2-k_Y^2}{k_Z^2-k_X^2}$ for angles of attack of 30°, 40°, 50°, 60°, and 70° are plotted in figures 19 to 23, inclusive. In these figures, as elsewhere in the report, negative sideslip is outward sideslip.

Values of aerodynamic yawing-moment coefficient which must be supplied by parts of the airplane other than the wings are plotted against the independent variables for angles of attack of 30°, 40°, 50°, 60°, and 70° in figures 24 to 28, inclusive. A negative value indicates that the moment supplied must oppose the rotation.

DISCUSSION

Purpose of including computations.—The computed values of sideslip and of yawing-moment coefficient for spinning equilibrium with a rectangular Clark Y wing are included as a basis for discussion of the meaning of the aerodynamic characteristics of the wing in terms of spinning characteristics. They also illustrate a comparatively simple and direct method of estimating the spinning characteristics of an airplane for which the inertia and aerodynamic characteristics are known or have been estimated.

Validity of assumptions.—The degree of approximation of the computed results caused by the approximations in the formulas has been discussed in the section giving the formulas. The approximations have no significant effect upon the points considered in the present discussion.

When making the computations, C_m was assumed to vary linearly with angle of attack and was not corrected for the effect of $\Omega b/2V$ or β. Such an assumption is legitimate for purposes of this discussion but should not be taken as other than very roughly representative of the case of an actual airplane.

It was assumed that no aerodynamic rolling moment is supplied by parts other than the wing or by interference effects. As a matter of fact, there will be rolling-moment contributions from the fuselage, horizontal and vertical tail surfaces, wheels, etc., but these will all be small in comparison with the wing rolling moment and will make but very minor changes in the estimated angle of sideslip.

When making the corrections in C_l and C_n, it was assumed that consistent differences that had been found between model and full-scale values (reference 8) were due to scale effects upon the wings. This assumption is probably correct for C_l but is not necessarily so for C_n, since the aerodynamic yawing moment from other parts of the airplane is of the same order of magnitude as that from the wings.

The values of ΔC_l and ΔC_n of 0.02 and 0.006, respectively, were chosen after consideration of the differences between model and full-scale data for tests in 7 different spinning conditions for an F4B–2 airplane and 2 different conditions for an NY–1 airplane. There is not sufficient experimental evidence to warrant placing implicit trust in the quantitative values of these corrections; but that corrections must be applied of the same order of magnitude seems beyond doubt, particularly in view of the fact that British investigators working under different conditions have found similar discrepancies between full-scale and model results (references 9 and 10).

Typical examples.—Typical cases showing the nature of the variations of C_l, β, and C_n with angle of attack are given. The first case (fig. 17) represents the mean condition assumed for the calculations and corresponds to $A=B$ with average values of C_m, μ, and $\frac{b^2}{k_Z^2-k_X^2}$. The second case (fig. 18) corresponds to the same values of C_m, μ, and $\frac{b^2}{k_Z^2-k_X^2}$ with $A>B$.

In both the illustrative cases, C_l varies from a small positive value at 30° angle of attack to a large negative value at 70°. All the curves of rolling moment equilibrium were similar. Positive values of C_l indicate that the outer wing tip is above the inner tip.

In all but one case the angle of sideslip necessary for equilibrium varied from a small value, either

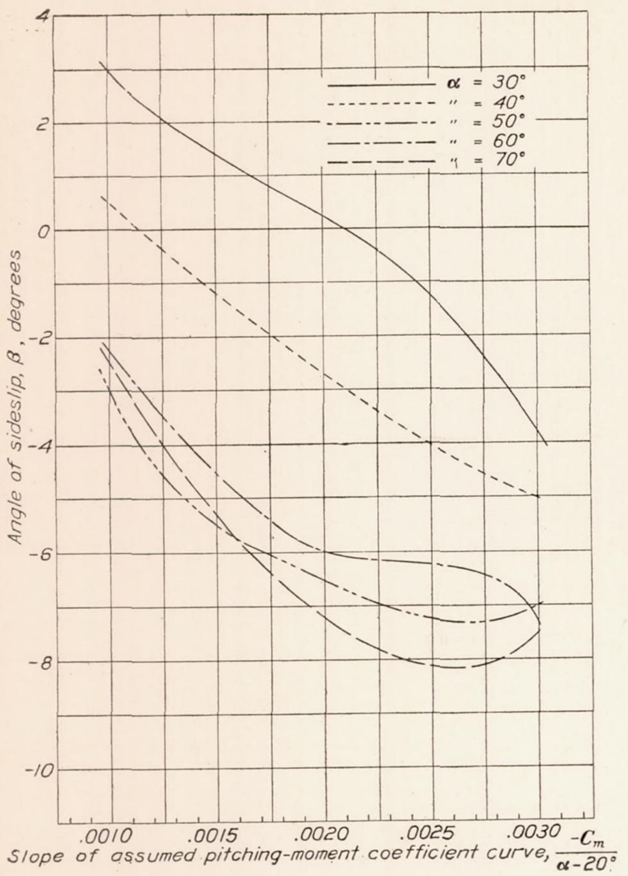

FIGURE 19.—Effect of pitching-moment coefficient upon sideslip necessary for equilibrium in a spin.

$\mu=5$ $\quad C_L=C_X''$ $\quad \frac{k_Z^2-k_Y^2}{k_Z^2-k_X^2}=1.0$ $\quad \frac{b^2}{k_Z^2-k_X^2}=80$

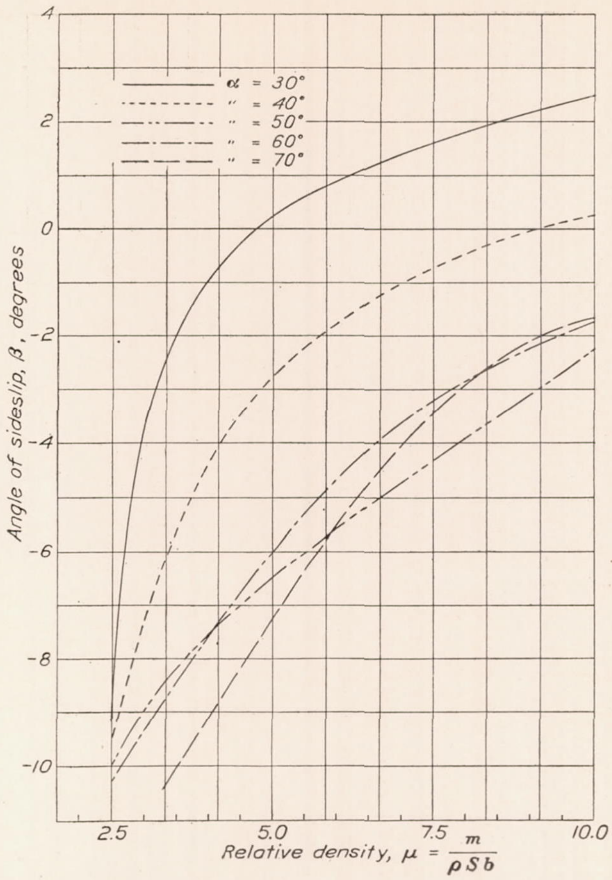

FIGURE 20.—Effect of relative density of airplane upon sideslip necessary for equilibrium in a spin.

$C_m=-0.0020(\alpha-20°)$ $\quad C_L=C_X''$ $\quad \frac{k_Z^2-k_Y^2}{k_Z^2-k_X^2}=1.0$ $\quad \frac{b^2}{k_Z^2-k_X^2}=80$

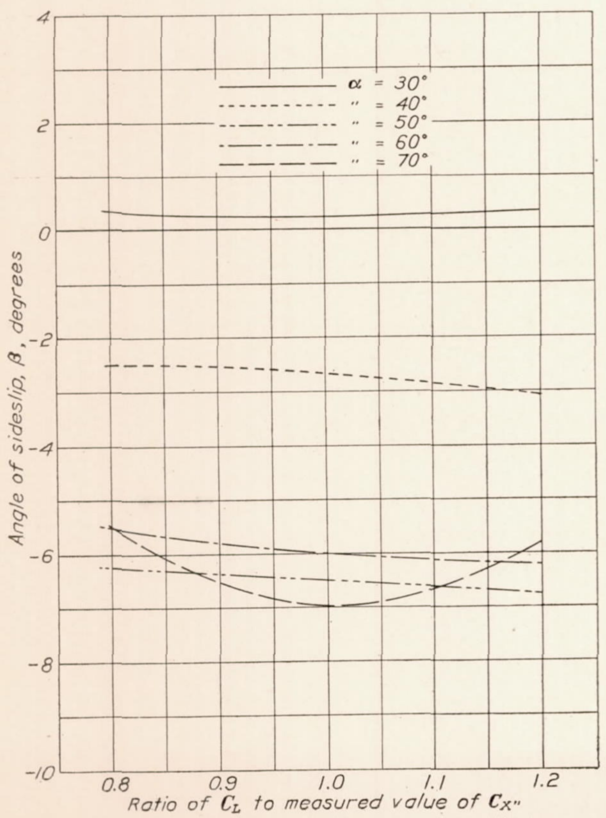

FIGURE 21.—Effect of lift coefficient upon sideslip necessary for equilibrium in a spin.

$\mu=5$ $\quad C_m=-0.0020(\alpha-20°)$ $\quad \frac{k_Z^2-k_Y^2}{k_Z^2-k_X^2}=1.0$ $\quad \frac{b^2}{k_Z^2-k_X^2}=80$

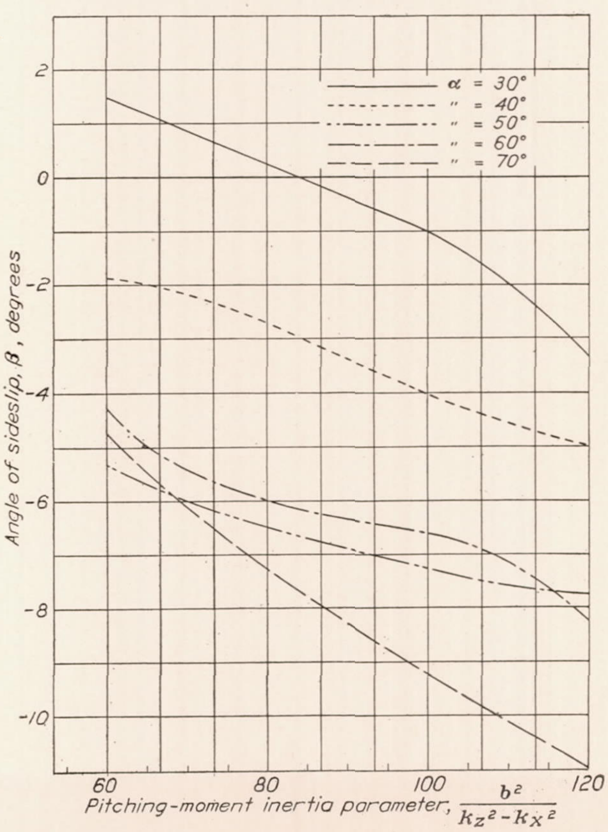

FIGURE 22.—Effect of pitching-moment inertia parameter upon sideslip necessary for equilibrium in a spin.

$\mu=5$ $\quad C_L=C_X''$ $\quad C_m=-0.0020(\alpha-20°)$ $\quad \frac{k_Z^2-k_Y^2}{k_Z^2-k_X^2}=1.0$

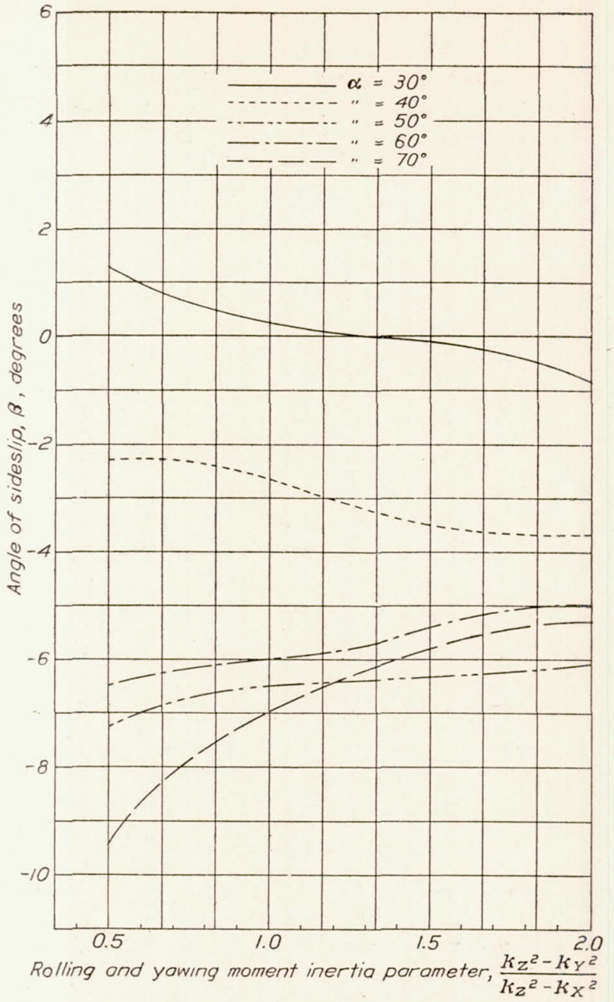

FIGURE 23.—Effect of rolling-moment and yawing-moment inertia parameter upon sideslip necessary for equilibrium in a spin.

$\mu=5$ $C_L=C_X''$ $C_m=-0.0020(\alpha-20°)$ $\frac{b^2}{k_Z^2-k_X^2}=80$

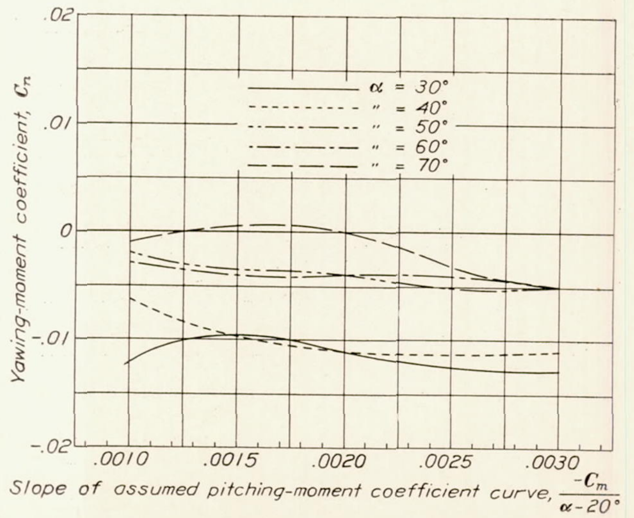

FIGURE 24.—Effect of pitching-moment coefficient upon yawing-moment coefficient that must be supplied by parts other than the wing for equilibrium in a spin.

$\mu=5$ $C_L=C_X''$ $\frac{k_Z^2-k_Y^2}{k_Z^2-k_X^2}=1.0$ $\frac{b^2}{k_Z^2-k_X^2}=80$

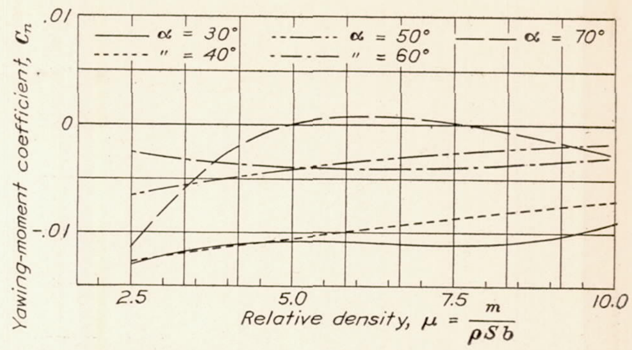

FIGURE 25.—Effect of relative density of airplane upon yawing-moment coefficient that must be supplied by parts other than the wing for equilibrium in a spin.

$C_m=-0.0020(\alpha-20°)$ $C_L=C_X''$ $\frac{k_Z^2-k_Y^2}{k_Z^2-k_X^2}=1.0$ $\frac{b^2}{k_Z^2-k_X^2}=80$

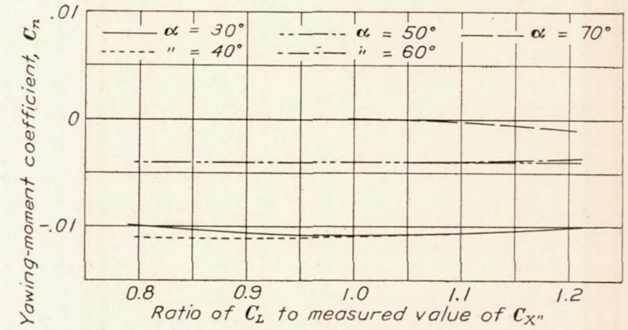

FIGURE 26.—Effect of lift coefficient upon yawing-moment coefficient that must be supplied by parts other than the wing for equilibrium in a spin.

$\mu=5$ $C_m=-0.0020(\alpha-20°)$ $\frac{k_Z^2-k_Y^2}{k_Z^2-k_X^2}=1.0$ $\frac{b^2}{k_Z^2-k_X^2}=80$

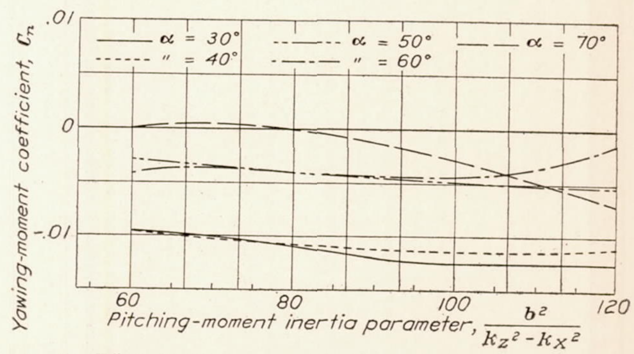

FIGURE 27.—Effect of pitching-moment inertia parameter upon yawing-moment coefficient that must be supplied by parts other than the wing for equilibrium in a spin.

$\mu=5$ $C_L=C_X''$ $C_m=-0.0020(\alpha-20°)$ $\frac{k_Z^2-k_Y^2}{k_Z^2-k_X^2}=1.0$

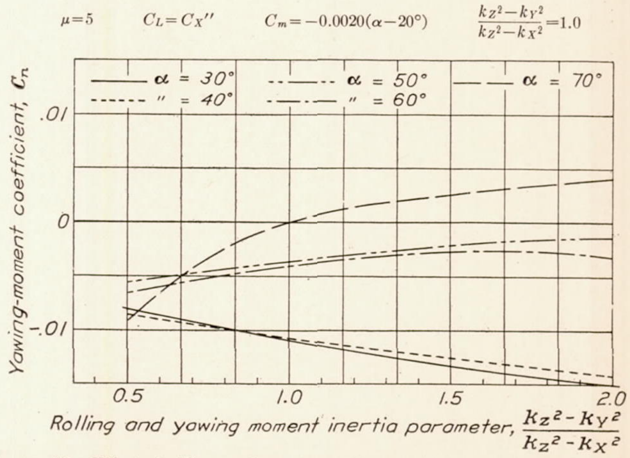

FIGURE 28.—Effect of rolling- and yawing-moment inertia parameter upon yawing-moment coefficient that must be supplied by parts other than the wing for equilibrium in a spin.

$\mu=5$ $C_L=C_X''$ $C_m=-0.0020(\alpha-20°)$ $\frac{b^2}{k_Z^2-k_X^2}=80$

inward or outward (positive or negative, respectively, for a right spin as here shown), at low angles of attack to a fairly large value outward at intermediate and high angles. The one exception was the case of a lightly loaded airplane ($\mu=2.5$) for which an outward sideslip of approximately 10° was required throughout the angle-of-attack range.

The variations of C_n with angle of attack shown in figures 17 and 18 are typical of all the computed values. The case shown in figure 17 represents the condition when $A=B$ and the inertia moment is therefore zero. Figure 18 illustrates the nature of the variation of the aerodynamic moment necessary to balance the inertia moment for a case when $A>B\left(\frac{k_Z^2-k_Y^2}{k_Z^2-k_X^2}>1\right)$. The values of C_n that must be supplied by the fuselage, empennage, and interference effects are in most cases negative and of the order of -0.01 at angles of attack of 30° and 40°, becoming less negative and in some cases slightly positive, at the higher angles.

Effect of variation of pitching-moment coefficient C_m.—Increasing the pitching-moment coefficient in the negative sense at any angle of attack increases the value of $\Omega b/2V$ necessary for equilibrium of pitching-moment coefficients and affects both the aerodynamic characteristics and the inertia rolling and yawing moments. As a result (see fig. 19) the amount of outward sideslip necessary for equilibrium of rolling moments increases as C_m is made more negative at any angle of attack. There was little change in the yawing-moment coefficient required for equilibrium (fig. 24) and no marked trend was revealed.

The net results of changing C_m will depend on the static stability in yaw (slope of curve of yawing moment against sideslip) of the airplane with respect to body axes at the angle of attack under consideration and upon the distance of the damping surfaces of the fuselage and empennage from the center of gravity. If it be assumed that the value of C_n from all parts other than the wing can be separated into one part due to the sideslip at the center of gravity and into another part due to the rotation, it can be shown that theoretically the yawing-moment coefficient due to sideslip varies linearly with sideslip and that the moment coefficient opposing the spin supplied by the damping surfaces increases approximately linearly with increase of $\Omega b/2V$. Such a treatment is valid for stability computations where small rates of rotation and small angles of sideslip are involved. The concept is valuable in considering the effect upon yawing-moment equilibrium in spins; but, because of the high rates of rotation and large angles of sideslip involved together with the accompanying interference effects, it is doubtful if very satisfactory results can be obtained from a quantitative analysis. On a qualitative basis it appears that increasing C_m in the negative sense will reduce the likelihood of attaining equilibrium in a spin if the airplane is statically stable in yaw because of the necessary increase in $\Omega b/2V$ and in outward sideslip. If the machine is unstable in yaw, the effect of attaining equilibrium depends on the relative magnitudes of the stability and the damping factors.

Effect of variation of relative density μ.—The smallest value of μ used in the analysis, corresponding to a wing loading of 6 pounds per square foot and a span of 31 feet, gave an indication of the necessity for very large values of outward sideslip for rolling equilibrium at all angles of attack. Increasing μ decreased the amount of outward sideslip very markedly, particularly at the lower angles of attack. It therefore appears that the airplane will spin with less outward sideslip when heavily loaded or at high altitudes than when lightly loaded or at low altitudes.

Increasing μ gave a general indication of decrease in the possibility of attaining yawing-moment equilibrium. The rate of change was generally small, although a large change was noted between $\mu=2.5$ and $\mu=5.0$ when α was 70°.

The net effect of increasing μ on the probability of attaining spinning equilibrium will depend on the static stability in yaw and upon the distance of damping elements from the center of gravity, as was pointed out in the discussion of the effect of pitching-moment equilibrium. As will be seen from equation (1), increasing μ decreases $\Omega b/2V$ and hence that portion of the yawing-moment coefficient due to rotation that always opposes the rotation. If the airplane is statically stable in yaw, increasing μ will tend to make easier the attainment of yawing-moment equilibrium by decreasing the moment opposing the spin because of the outward sideslip. The change in value of C_n necessary with increase of μ will also affect the result. In general, it is probable that increasing μ will increase the possibility of attaining spinning equilibrium.

Effect of changing C_L.—There were no changes worthy of note with change in C_L.

Effect of changing pitching-moment inertia parameter $\frac{b^2}{k_Z^2-k_X^2}$.—Increasing this parameter while keeping $\frac{k_Z^2-k_Y^2}{k_Z^2-k_X^2}$ constant is equivalent to decreasing both A and C with respect to Wb^2. Such a change results in the necessity for increased amounts of outward sideslip at all angles of attack. The effect on yawing-moment coefficient required for balance was slight, although there was a general tendency toward increase of likelihood of attaining equilibrium except at 60° angle of attack.

The net effect of changes in $\frac{b^2}{k_Z^2-k_X^2}$ upon the possibility of attaining spinning equilibrium is similar to that of changes in C_m since increasing the parameter also increases the value of $\Omega b/2V$.

Effect of changing the rolling-moment and yawing-moment inertia parameter $\frac{k_Z^2-k_Y^2}{k_Z^2-k_X^2}$.—When $\frac{k_Z^2-k_X^2}{k_Z^2-k_Y^2}=1$, $B=A$ and the gyroscopic yawing moment is zero. (See equation (3).) Increasing the parameter without changing $\frac{b^2}{k_Z^2-k_X^2}$ is equivalent to decreasing B or to decreasing the mass distribution along the longitudinal axis with respect to that along the lateral axis. Such a change resulted in the requirement of slightly more outward sideslip for equilibrium at low angles of attack and less outward sideslip at high angles of attack. Increasing this parameter also resulted in an increase in the probability of attaining yawing-moment equilibrium at low angles of attack and a decrease in the possibility of attaining that equilibrium at high angles of attack. This effect is largely due to the change in the gyroscopic yawing-moment coefficient.

Since changes in $\frac{k_Z^2-k_Y^2}{k_Z^2-k_X^2}$ do not affect $\Omega b/2V$ the net effect upon the possibility of attaining spinning equilibrium will depend upon the effect on sideslip and the effect on C_n required for equilibrium. If the airplane is statically stable in yaw, increasing the ratio will produce counteracting effects. If the airplane is statically unstable in yaw, the effects will be additive, making very much more likely the attainment of equilibrium at low angles of attack and very much less likely the attainment of equilibrium at high angles.

Effect of ΔC_l and ΔC_n on the computed results.—The discussion of the effect upon the likelihood of securing spinning equilibrium has been confined to cases for which the values for C_l and C_n obtained from model tests have been changed by $\Delta C_l=0.02$ and $\Delta C_n=0.006$, respectively. If the value of C_l obtained from model tests had been used directly, 2.5° to 7.5° more outward sideslip than shown in figures 17 to 28 would have been needed for equilibrium of rolling moments.

As sideslip has but small influence upon the aerodynamic value of C_n, the effect of omitting ΔC_l upon the value of C_n necessary for equilibrium would have been small for the cases with the parameter $\frac{k_Z^2-k_Y^2}{k_Z^2-k_X^2}=1$. Had this ratio been other than 1, the effect would have been to give the curves of C_n required at low angles of attack the same slope as those in the present analysis at high angles of attack, and to show an even less possibility of obtaining equilibrium in high-angle-of-attack spins with increase of the ratio.

Adding $\Delta C_n=0.006$ to the measured values had no other effect than to add -0.006 to the value of C_n required of the fuselage, empennage, and interference effects for equilibrium of yawing moments.

It is apparent that, if the foregoing corrections represent the true differences between model and full-scale results, models can be expected to spin with about 5° more outward sideslip than the full-scale airplane and to be not so likely to attain steady spinning equilibrium at a given angle of attack if statically stable in yaw. The evidence is insufficient, however, to substantiate fully a conclusion to that effect.

CONCLUSIONS

If it be assumed that the corrections applied are of the right order of magnitude, the following conclusions are indicated by the analysis presented for a conventional monoplane with a rectangular Clark Y wing:

1. The value of the yawing-moment coefficient required from the fuselage, tail, and interference effects for steady spinning equilibrium is small throughout the angle-of-attack range investigated. It appears that the spinning attitude of such an airplane will depend very greatly upon details of arrangement of the fuselage and tail.

2. The maximum yawing-moment coefficient that must be supplied by parts other than the wing to insure recovery from steady spinning equilibrium is $C_n=-0.02$.

3. Increasing the static stability in yaw (making more negative the slope of curve of yawing moment against sideslip) about body axes at spinning angles of attack will decrease the possibility of attaining equilibrium of yawing moments at angles of attack greater than 40° and hence will tend to prevent flat spins.

Langley Memorial Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Langley Field, Va., *November 21, 1934.*

APPENDIX

EQUATIONS FOR USE IN COMPUTING STEADY SPINNING EQUILIBRIUM

Derivation.—If it be assumed that the cross-wind force in a steady spin without sideslip is zero, the equations of balance of forces read,

$$\frac{1}{2}\rho V^2SC_L \cos \psi' \cos \varphi = m\Omega^2 R \tag{4}$$

$$\frac{1}{2}\rho V^2SC_D \cos \sigma + \frac{1}{2}\rho V^2SC_L \sin \varphi' = \mathrm{mg} \tag{5}$$

$$\frac{1}{2}\rho V^2SC_D \sin \sigma = \frac{1}{2}\rho V^2SC_L \sin \psi' \cos \varphi' \tag{6}$$

where R is the radius of the spin,

ψ', the angle between the projection of the lift vector upon a horizontal plane and the spin radius.

φ', the angle between the horizontal and the lift vector.

and σ, the angle between the flight path and the vertical.

The foregoing equations of balance of forces are true regardless of sideslip if the cross-wind force is such that

$$C_c = -C_R \sin \cos^{-1} (\cos^2 \alpha + \sin^2 \alpha \cos \beta)$$

$$\sin \cos^{-1} (\cos^2 \alpha + \sin^2 \alpha \cos \beta) = \sin \beta \sin \alpha \text{ (approx.)}$$

It can be shown that φ is the $\tan^{-1}$ ($\tan \sigma \sin \psi'$). Both σ and ψ' are generally small and φ must necessarily be quite small so that $\sin \varphi$ may be considered 0 and $\cos \varphi$ taken equal to 1.

Also, since

$$\sin \sigma = \frac{\Omega R}{V} \tag{7}$$

and

$$\cos \sigma = \frac{\sqrt{V^2 - \Omega R^2}}{V} \tag{8}$$

$$\frac{1}{2}\rho V^2SC_L \cos \psi' = m\Omega^2 R \tag{9}$$

$$\frac{1}{2}\rho V^2SC_D \frac{\sqrt{V^2 - \Omega R^2}}{V} = \mathrm{mg} \tag{10}$$

$$C_D \frac{\Omega R}{V} = C_L \sin \psi' \tag{11}$$

From the necessity for balance of the gyroscopic and aerodynamic pitching moments,

$$\frac{1}{2}\rho V^2SbC_m = (A-C)pr$$
$$= (A-C)\Omega^2 \cos \alpha \sin \alpha \cos^2 (\sigma+\beta) \text{(approx.)} \tag{12}$$

The angle $(\sigma+\beta)$ is approximately the angle of the Y axis to the horizontal and is generally less than 10° and seldom greater than 20°.

Assuming that an average value of $(\sigma+\beta)$ is 11°, $\cos^2 (\sigma+\beta) = 0.96$ and (12) may be rewritten as

$$\frac{1}{2}\rho V^2SbC_m = 0.96\Omega^2(A-C) \sin \alpha \cos \alpha \tag{13}$$

Eliminating $\sin \psi'$ and $\cos \psi'$ from equations (9) and (11) and substituting from equation (13),

$$R = \frac{-0.96(A-C) \sin \alpha \cos \alpha \, C_L}{\sqrt{m^2b^2C_m{}^2 - C_D{}^2 \frac{1}{2}\rho SbC_m \times 0.96(A-C) \sin \alpha \cos \alpha}} \tag{14}$$

Experience with actual values has shown that the second term under the radical sign is negligible compared with the first and therefore equation (14) may be rewritten,

$$R = \frac{-0.96(A-C) \sin 2\alpha C_L}{mbC_m} \tag{15}$$

Also

$$\frac{1}{2}\rho V^2SbC_l = \Omega^2 \sin \alpha \sin (\sigma+\beta) \cos (\sigma+\beta) \text{ (C}-\text{B) (nearly)} \tag{16}$$

and

$$\frac{1}{2}\rho V^2SbC_n = \Omega^2 \cos \alpha \sin (\sigma+\beta) \cos (\sigma+\beta) (B-A) \text{ (nearly)} \tag{17}$$

or for an average case,

$$\frac{1}{2}\rho V^2SbC_l = 0.98\Omega^2 \sin \alpha \sin (\sigma+\beta) (C-B) \tag{18}$$

$$\frac{1}{2}\rho V^2SbC_n = 0.98\Omega^2 \cos \alpha \sin (\sigma+\beta) (B-A) \tag{19}$$

If it be further assumed that

$$\sin (\sigma+\beta) = \sin \sigma + \sin \beta$$

it follows that,

$$C_l = \frac{(C-B)C_L}{m} \sqrt{\frac{0.48 \rho \tan \alpha C_m}{b(A-C)}} + \frac{(C-B)}{(A-C)} \frac{C_m \sin \beta}{\cos \alpha} \tag{20}$$

Letting $\mu = \frac{m}{\rho Sb}$ the equations finally resulting are,

$$\frac{\Omega b}{2V} = \sqrt{\frac{-C_m}{3.84 \mu \sin 2\alpha} \times \frac{b^2}{k_Z{}^2 - k_X{}^2}} \tag{21}$$

$$C_l = C_L\left(\frac{k_Z^2 - k_Y^2}{b\sqrt{k_Z^2 - k_X^2}}\right)\sqrt{\frac{-C_m \tan\alpha}{2\mu}} + 1.02\left(\frac{k_Z^2 - k_Y^2}{k_Z^2 - k_X^2}\right)\frac{-C_m \sin\beta}{\cos\alpha} \qquad (22)$$

and

$$C_n = C_l \cot\alpha\left(\frac{k_Y^2 - k_X^2}{k_Z^2 - k_Y^2}\right) \qquad (23)$$

Discussion.—The foregoing equations were developed from a study of the geometry of the spin along the lines suggested in reference 11. They differ from the equations in regular use by British investigators (reference 1) in the following particulars.

In the foregoing derivation, cross-wind force is assumed to vary in a specific manner with sideslip rather than to be zero. The angle ψ' is eliminated by trigonometrical substitution rather than by treating $\cos\psi$ as equal to 1. These differences make no essential difference in the results.

British investigators have found it necessary to obtain the total rolling-moment coefficient when the wing is rolling and sideslipping by adding to the rolling-moment coefficient when rolling without sideslip a value of rolling-moment coefficient obtained with the wing sideslipping but not rolling. Their equation for determination of the angle of sideslip for equilibrium (reference 1) is based on this assumption. With the N. A. C. A. spinning balance the rolling-moment coefficient can be obtained with the wing rolling and sideslipping. The foregoing equations were developed to permit of a graphical solution for angle of sideslip using the spinning-balance data.

The foregoing equations retain small correction factors that arise from consideration of the cosines of angles that may vary from 0° to 20° as having an average value of 0.98.

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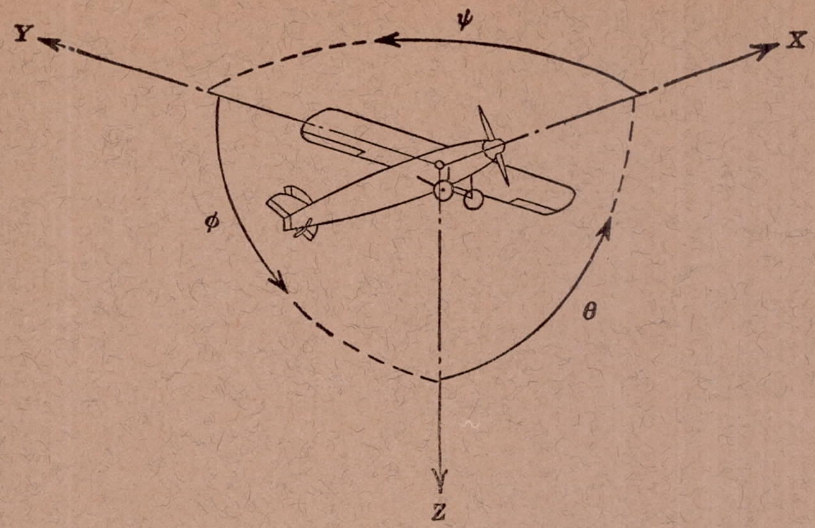

Positive directions of axes and angles (forces and moments) are shown by arrows

Axis		Force (parallel to axis) symbol	Moment about axis			Angle		Velocities	
Designation	Symbol		Designation	Symbol	Positive direction	Designation	Symbol	Linear (component along axis)	Angular
Longitudinal	X	X	Rolling	L	$Y \longrightarrow Z$	Roll	ϕ	u	p
Lateral	Y	Y	Pitching	M	$Z \longrightarrow X$	Pitch	θ	v	q
Normal	Z	Z	Yawing	N	$X \longrightarrow Y$	Yaw	ψ	w	r

Absolute coefficients of moment

$C_l = \frac{L}{qbS}$ (rolling) $C_m = \frac{M}{qcS}$ (pitching) $C_n = \frac{N}{qbS}$ (yawing)

Angle of set of control surface (relative to neutral position), δ. (Indicate surface by proper subscript.)

4. PROPELLER SYMBOLS

D, Diameter
p, Geometric pitch
p/D, Pitch ratio
V', Inflow velocity
V_s, Slipstream velocity
T, Thrust, absolute coefficient $C_T = \frac{T}{\rho n^2 D^4}$
Q, Torque, absolute coefficient $C_Q = \frac{Q}{\rho n^2 D^5}$
P, Power, absolute coefficient $C_P = \frac{P}{\rho n^3 D^5}$
C_s, Speed-power coefficient $= \sqrt[5]{\frac{\rho V^5}{P n^2}}$
η, Efficiency
n, Revolutions per second, r.p.s.
Φ, Effective helix angle $= \tan^{-1}\left(\frac{V}{2\pi r n}\right)$

5. NUMERICAL RELATIONS

1 hp. = 76.04 kg-m/s = 550 ft-lb./sec.
1 metric horsepower = 1.0132 hp.
1 m.p.h. = 0.4470 m.p.s.
1 m.p.s. = 2.2369 m.p.h.
1 lb. = 0.4536 kg.
1 kg = 2.2046 lb.
1 mi. = 1,609.35 m = 5,280 ft.
1 m = 3.2808 ft.